# Simple Linear Regression

## Proof of the Closed Form Solution

Rahul Singh
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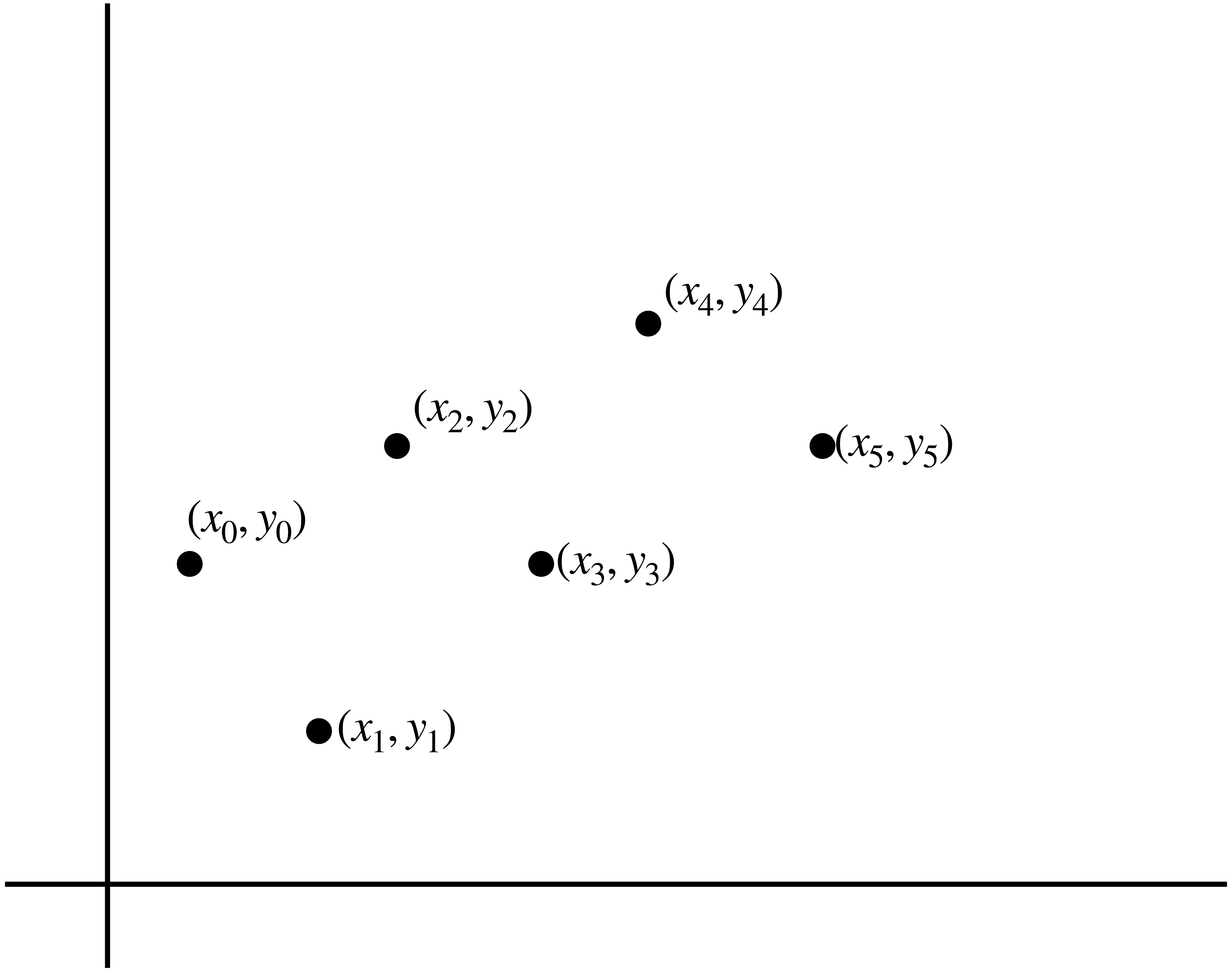
## Problem Statement

Given a set of data points in $\mathbb{R}^2$, $(x_0, y_0), (x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$, find the line that best fits the data



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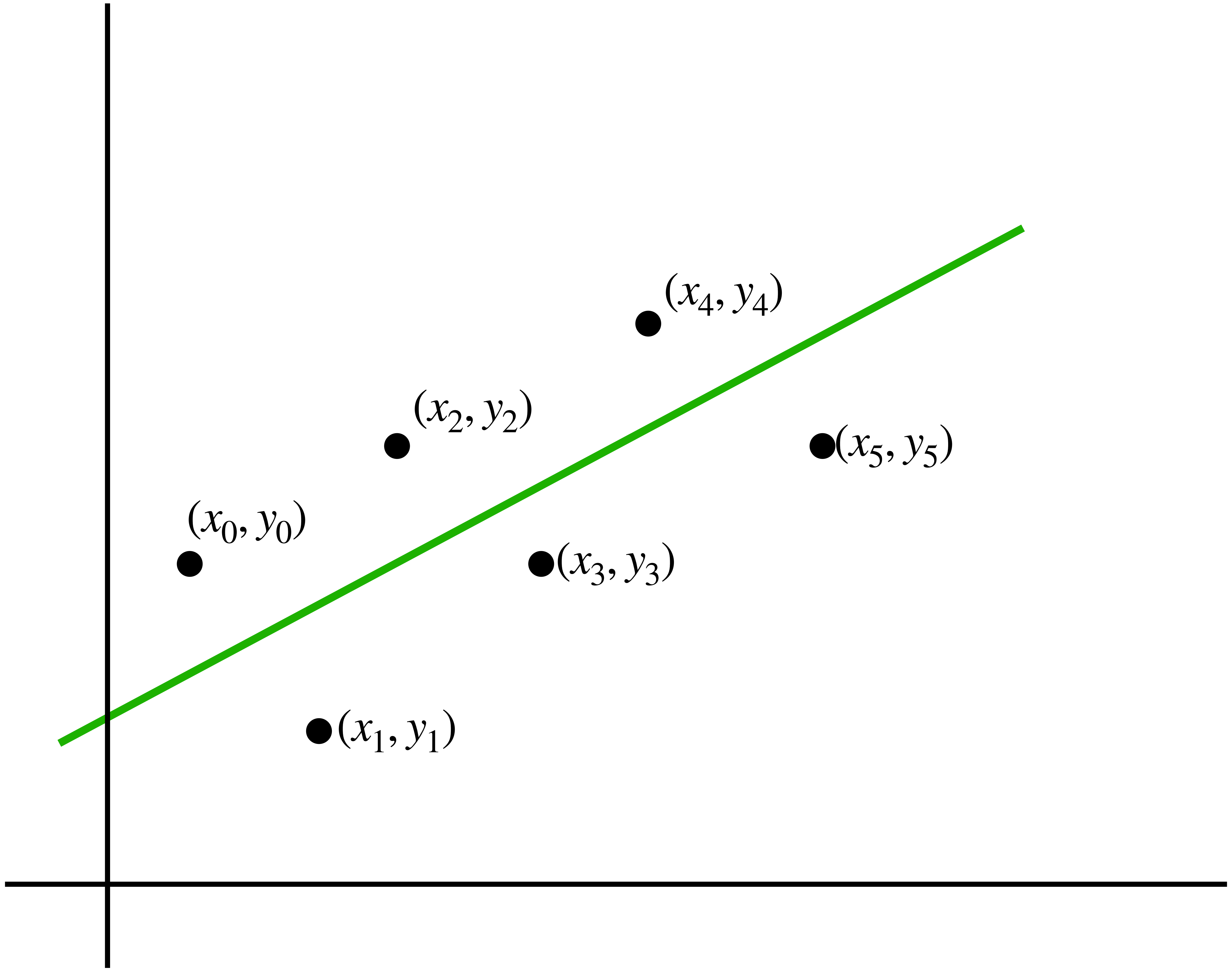

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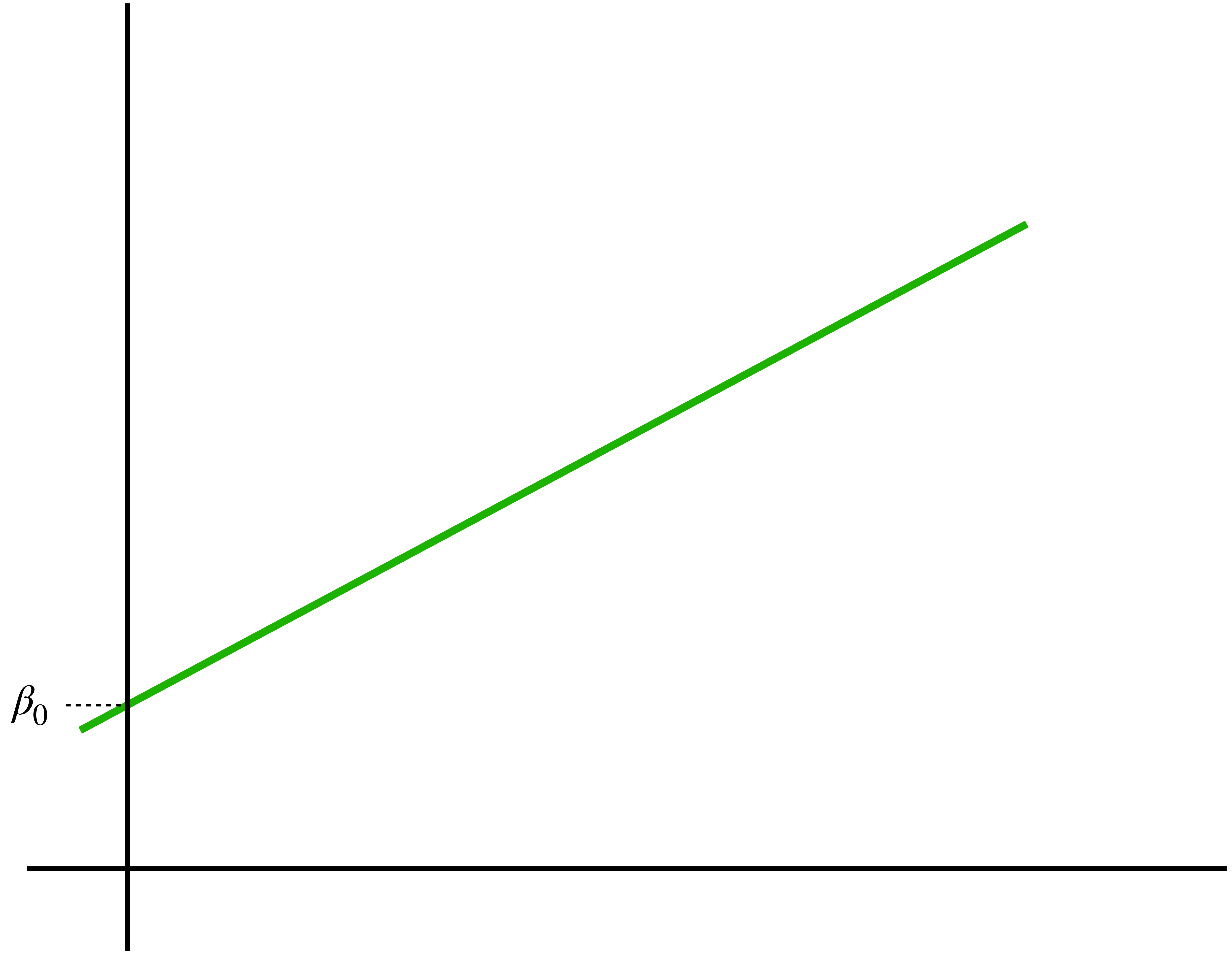

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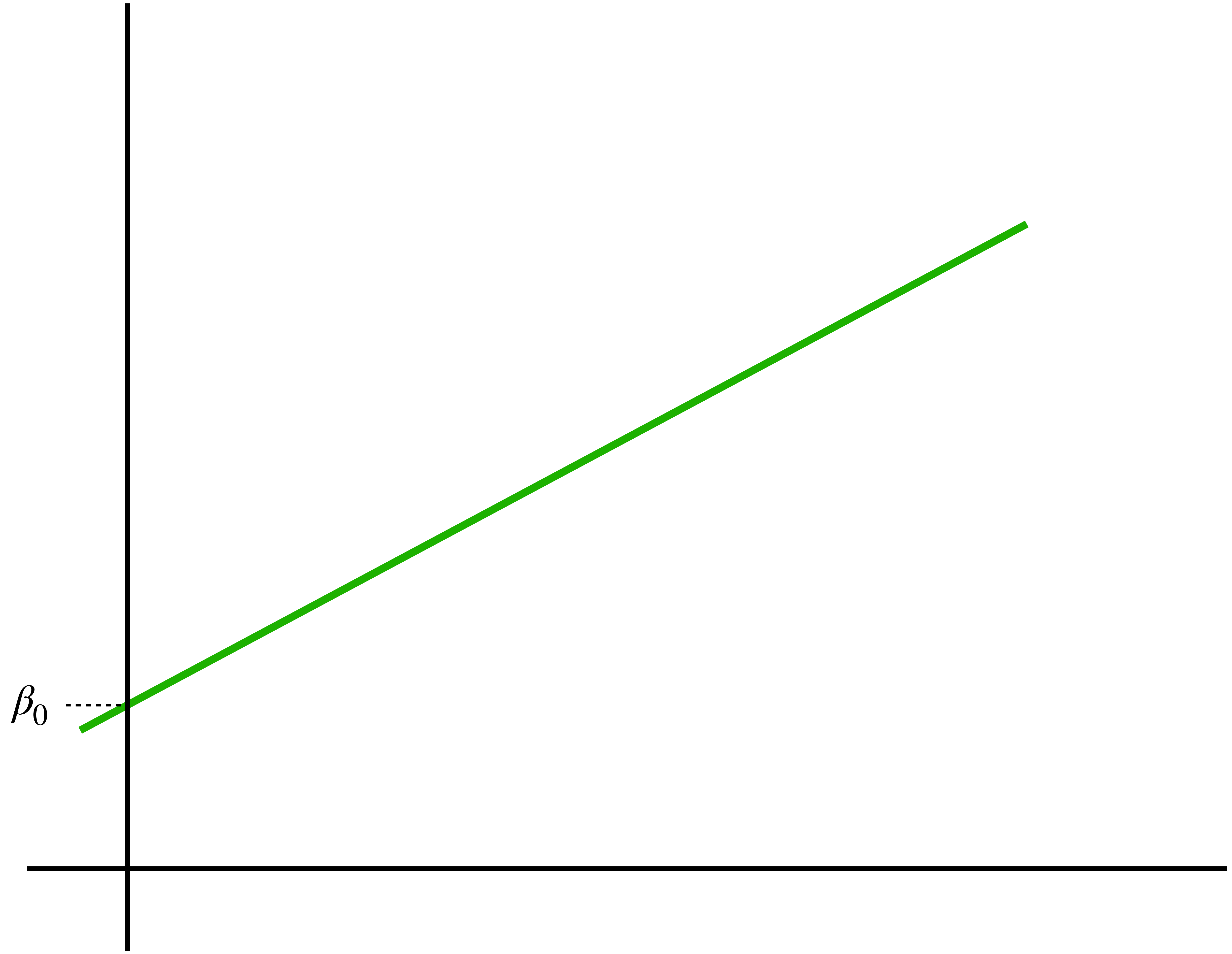

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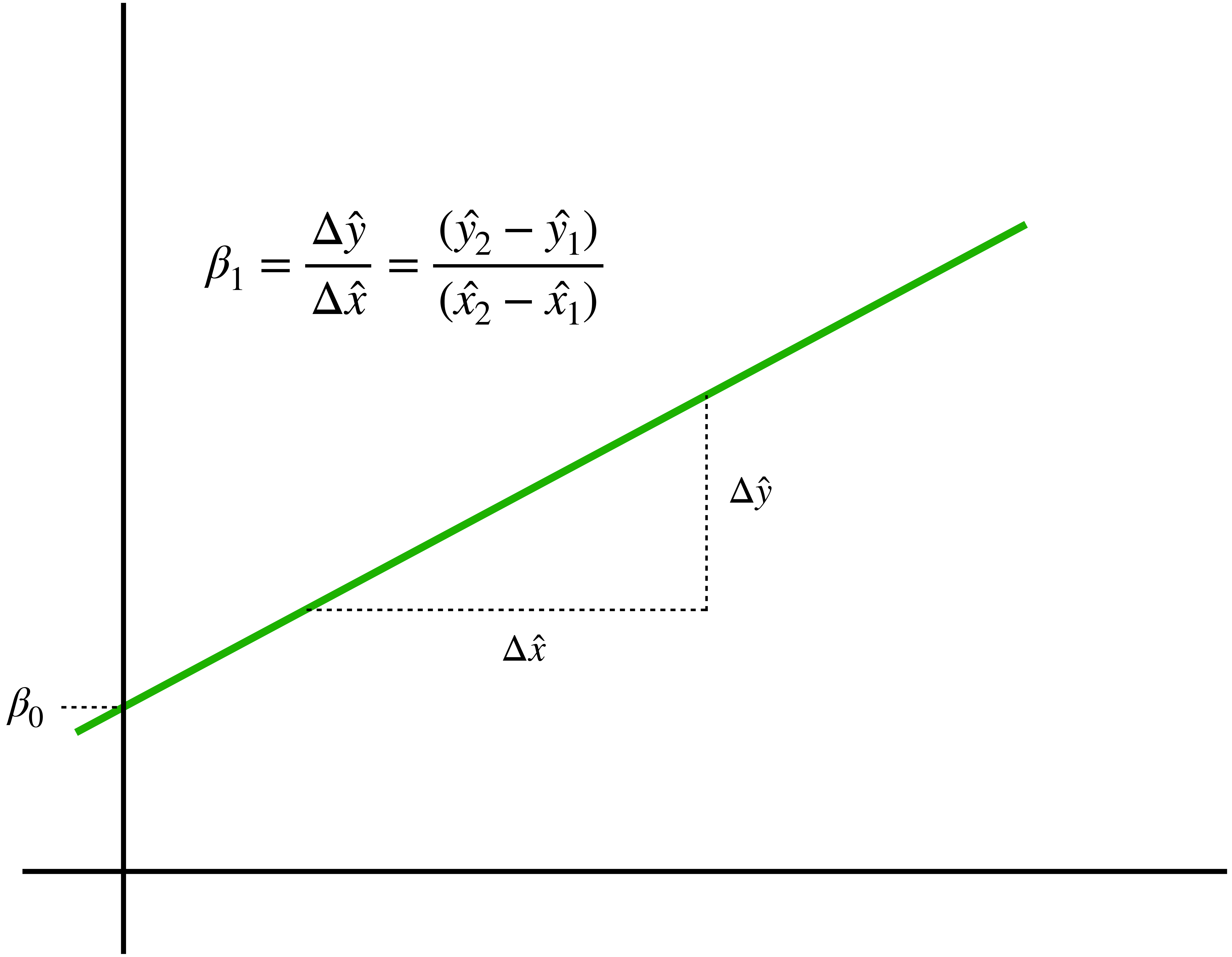

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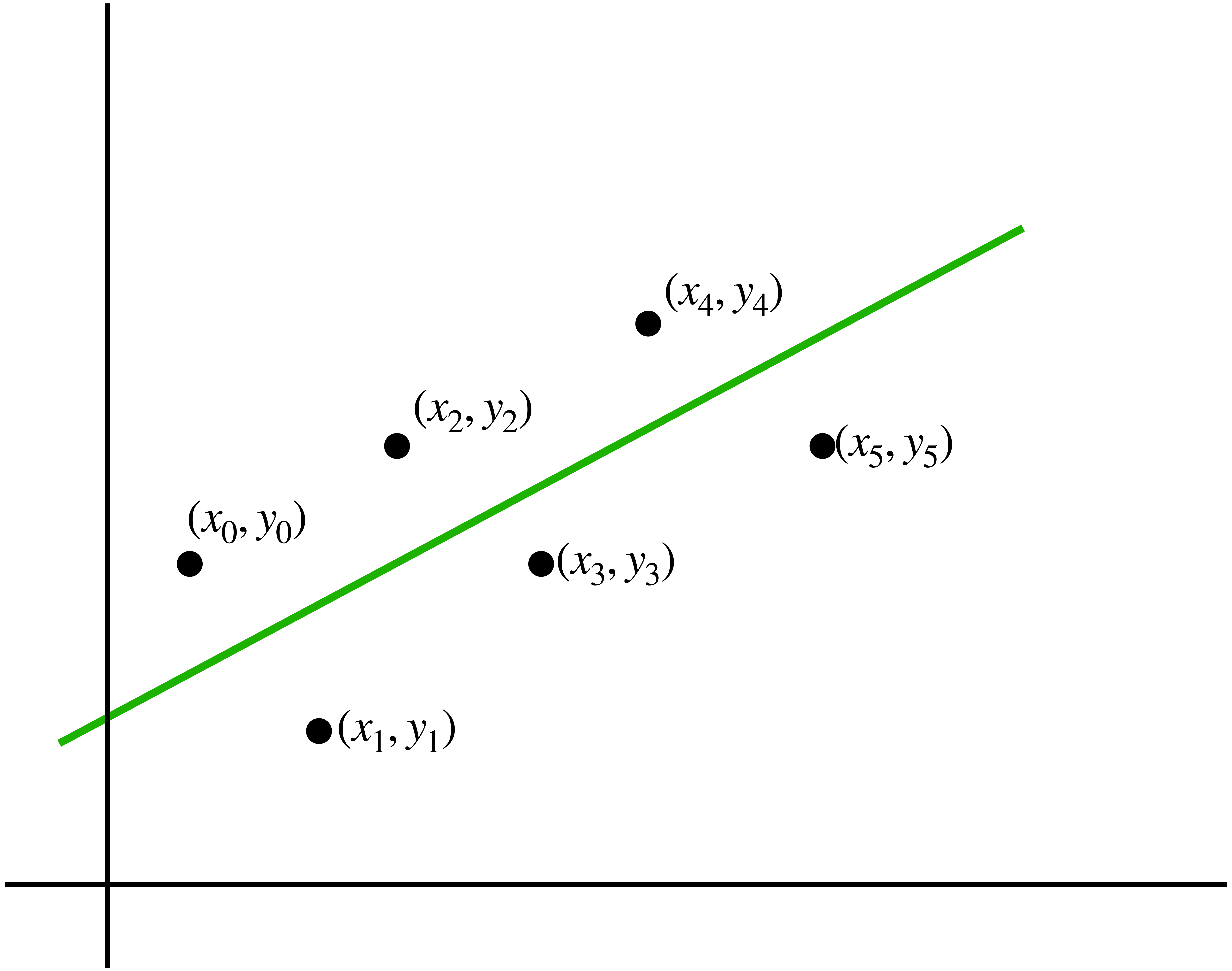

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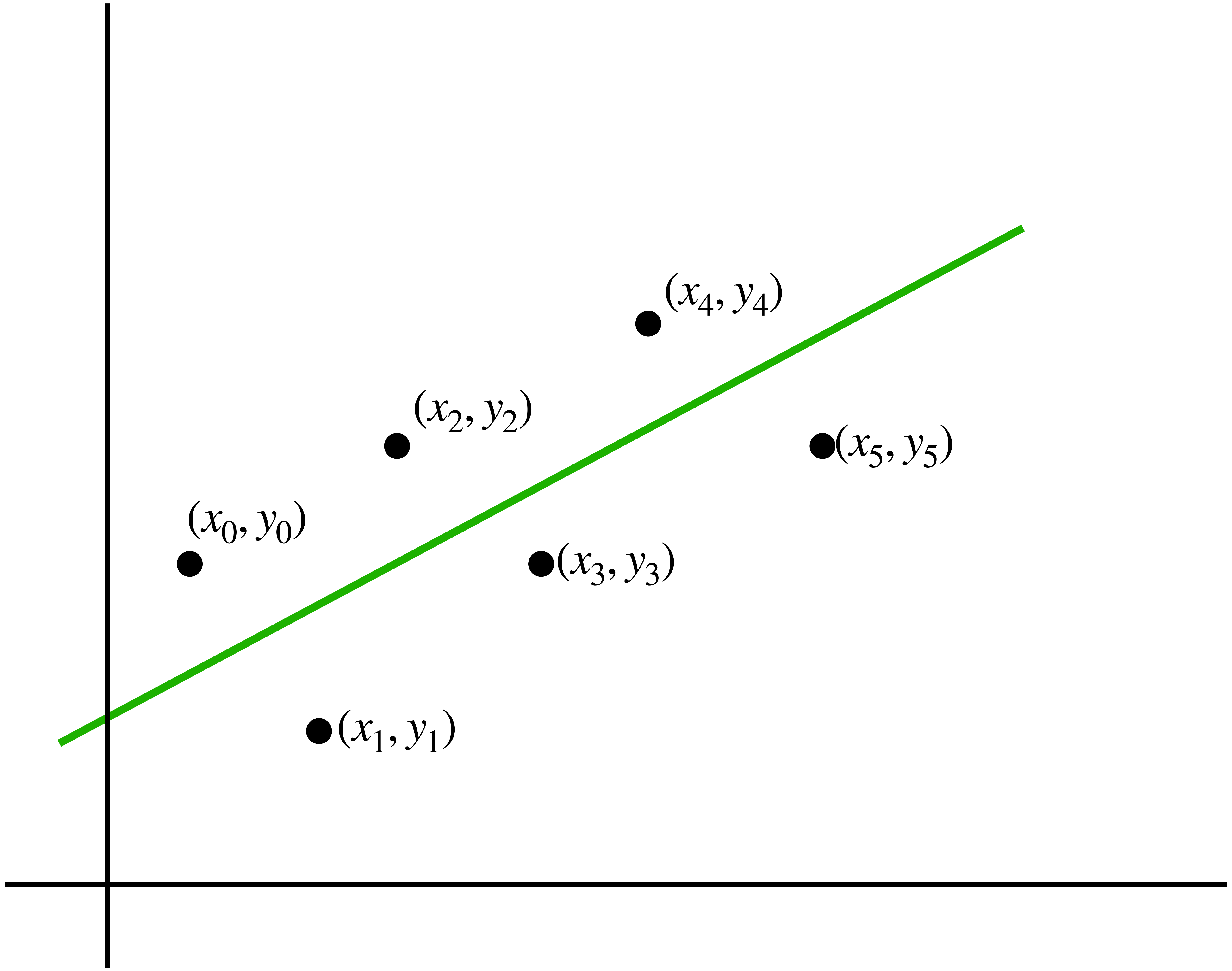

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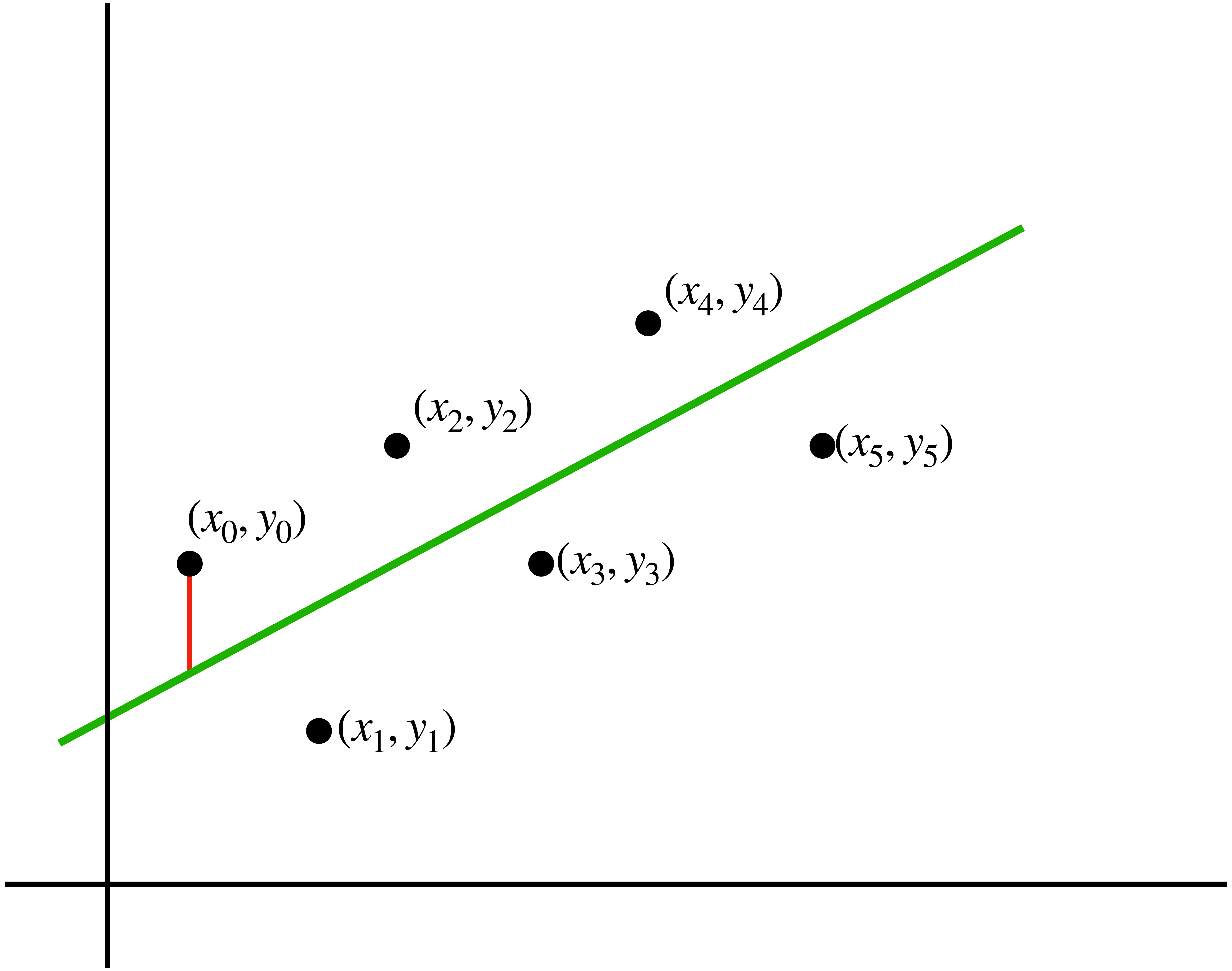

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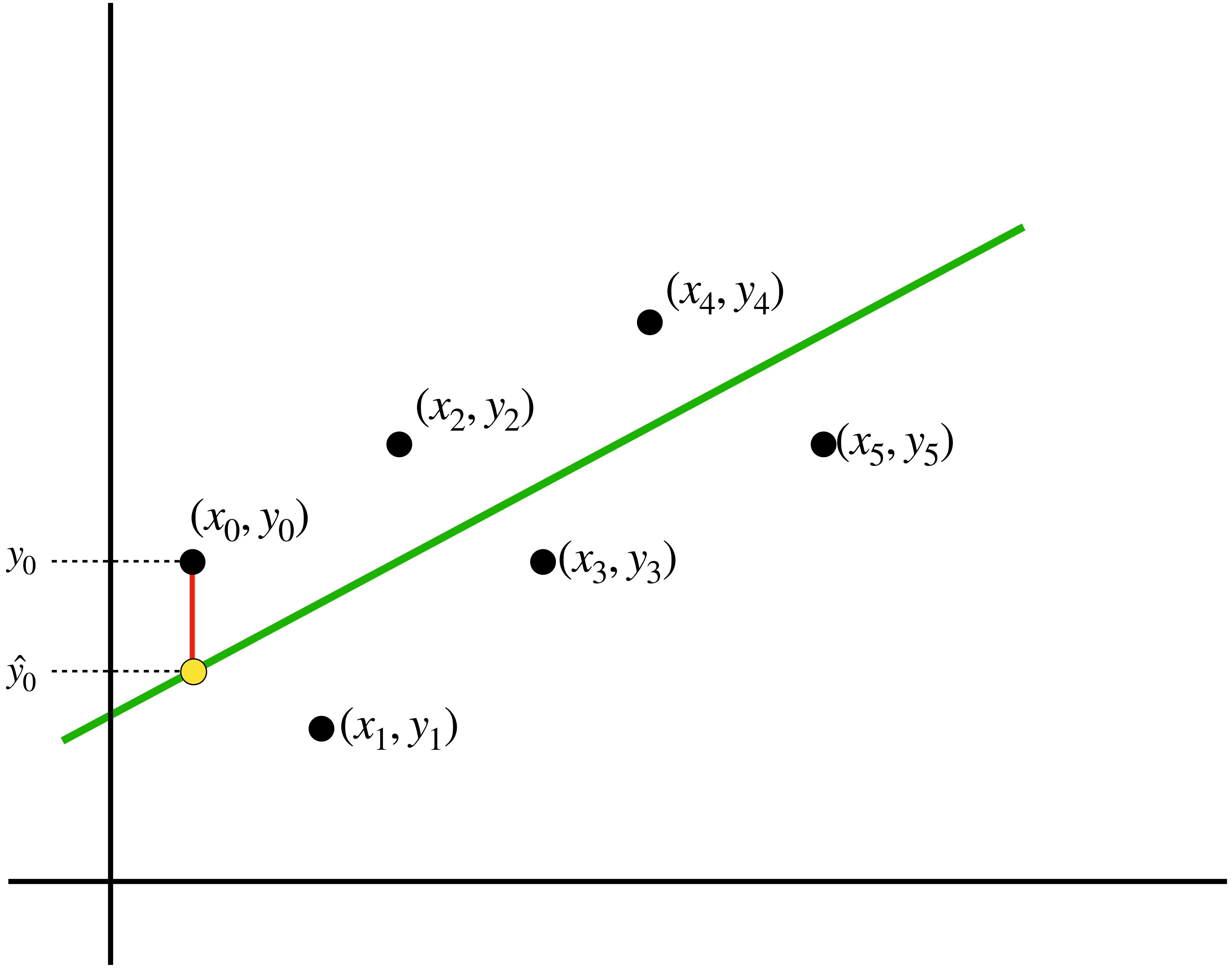

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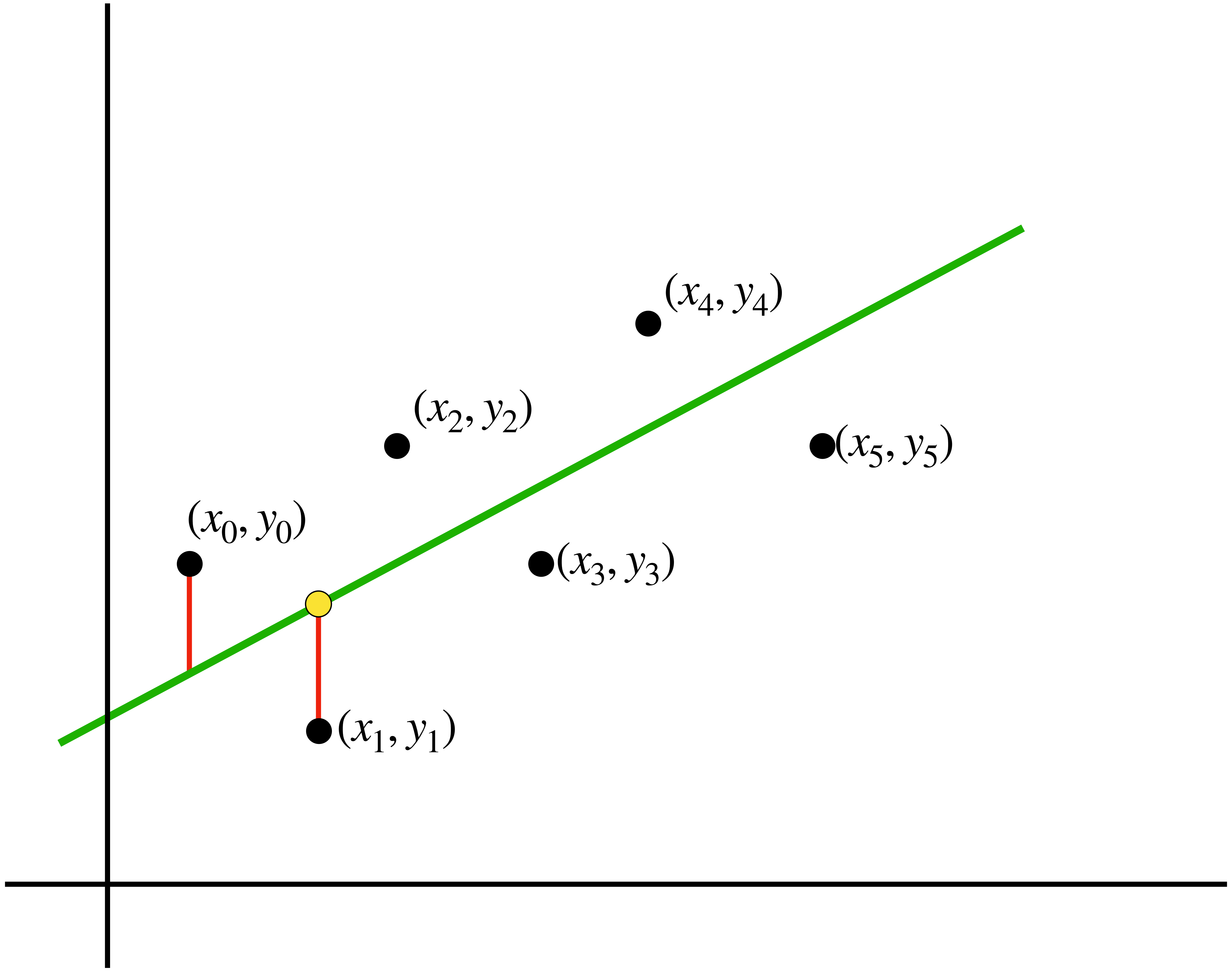

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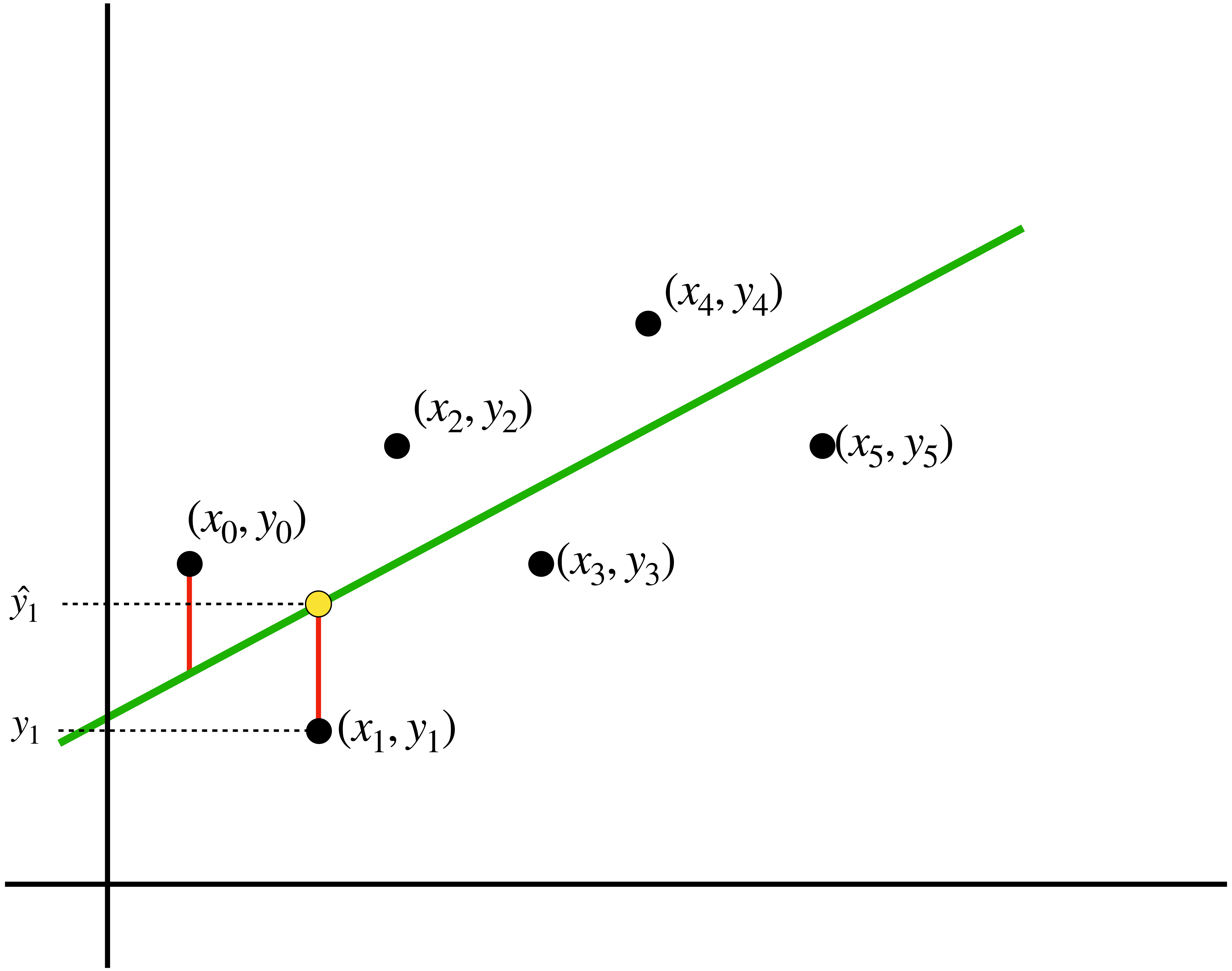

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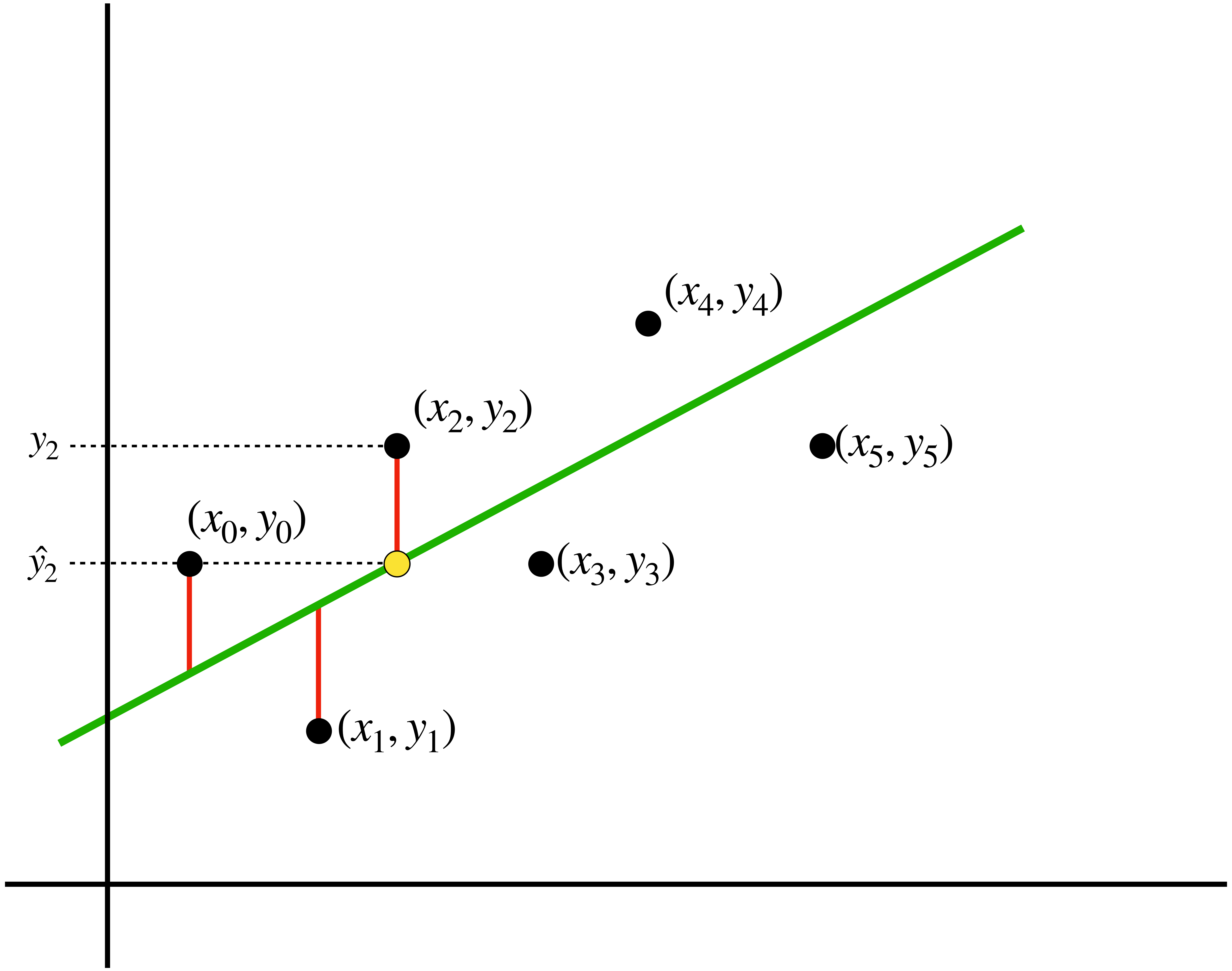


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$$(y_0 - \hat{y}_0)^2 + (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2$$
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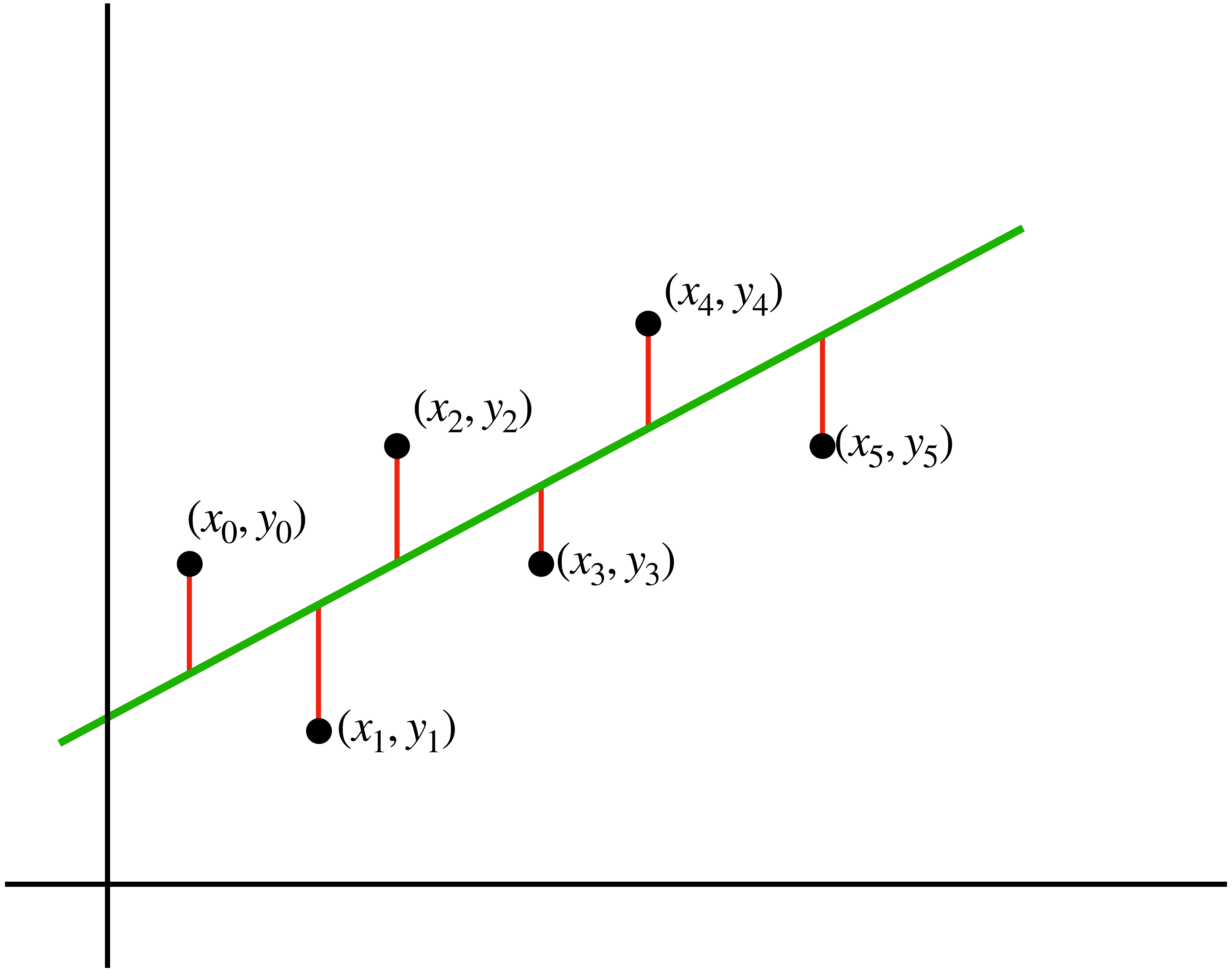

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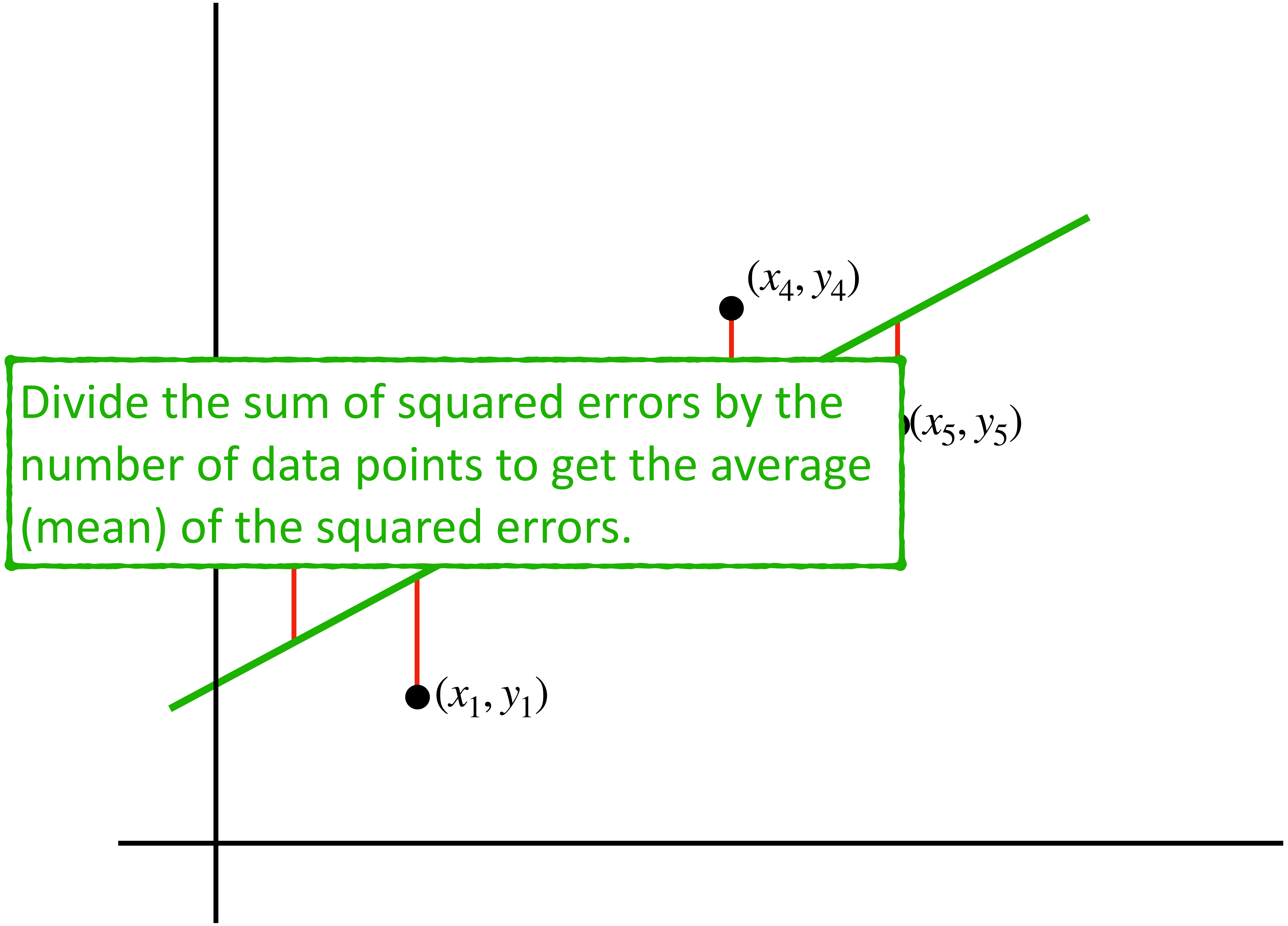
Mean Squared Error (MSE):

$$\frac{(y_0 - \hat{y}_0)^2 + (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + (y_3 - \hat{y}_3)^2 + (y_4 - \hat{y}_4)^2 + (y_5 - \hat{y}_5)^2}{n}$$

$$\frac{1}{n} \sum_{i=0}^{n} (y_i - \hat{y}_i)^2$$



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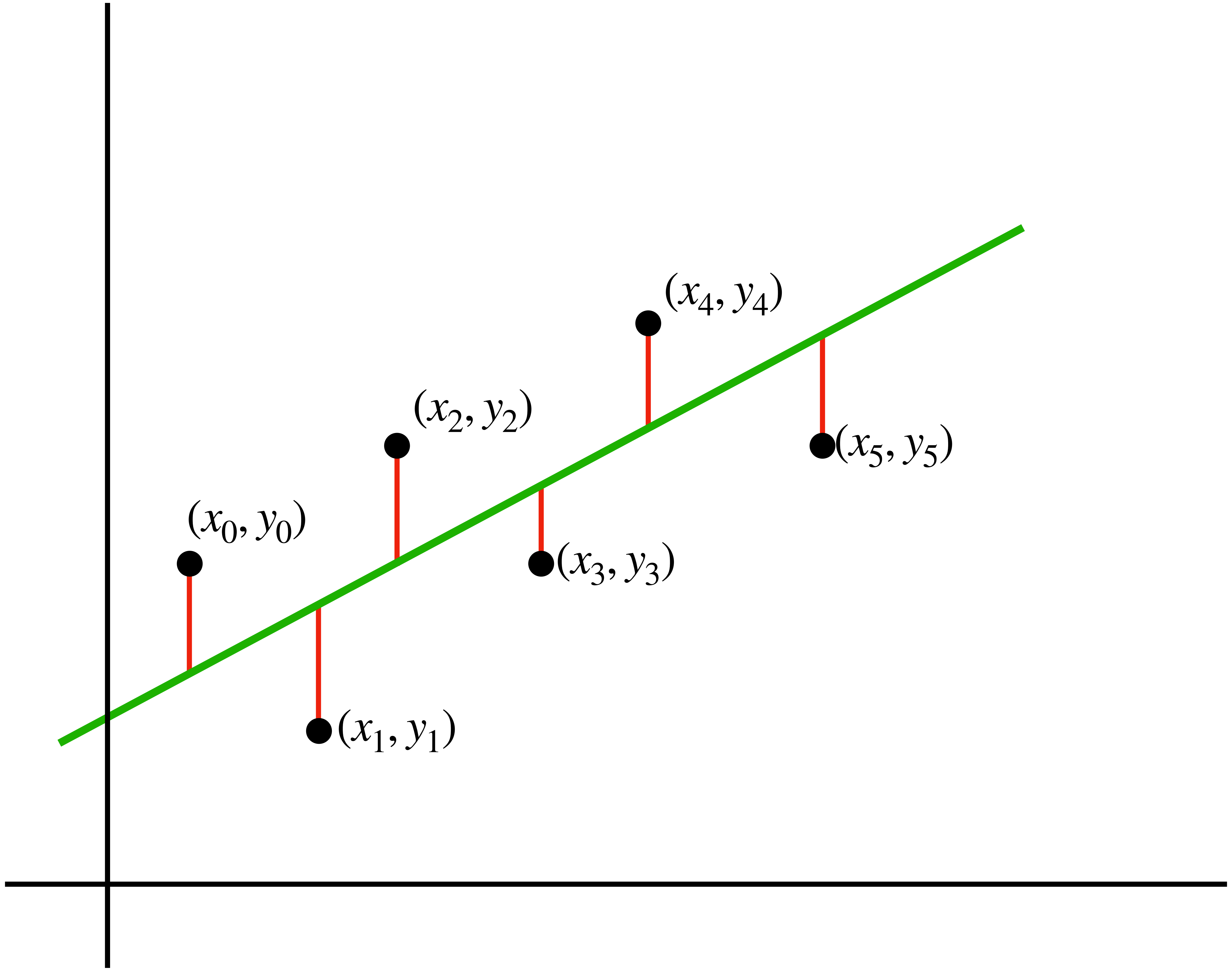

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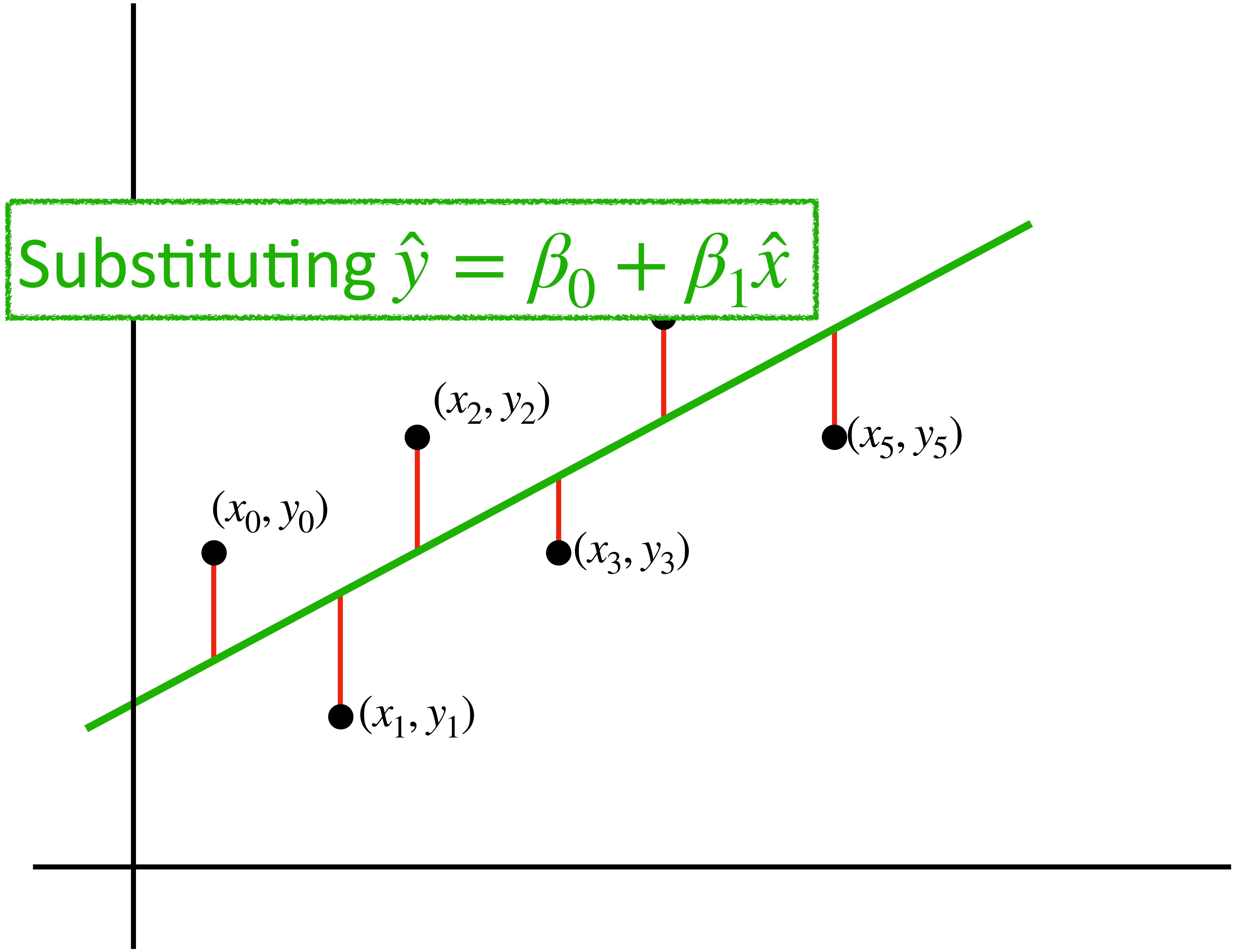

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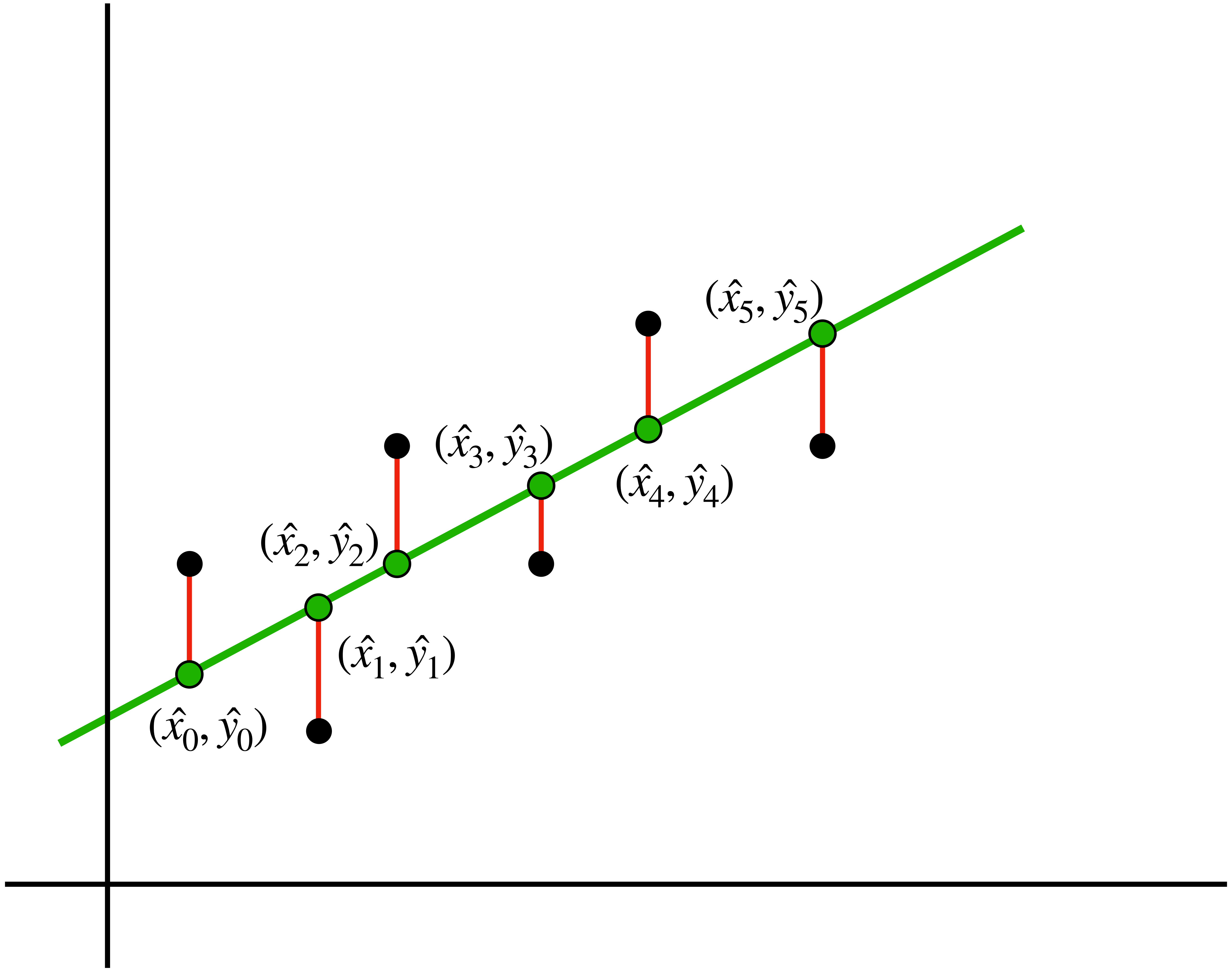

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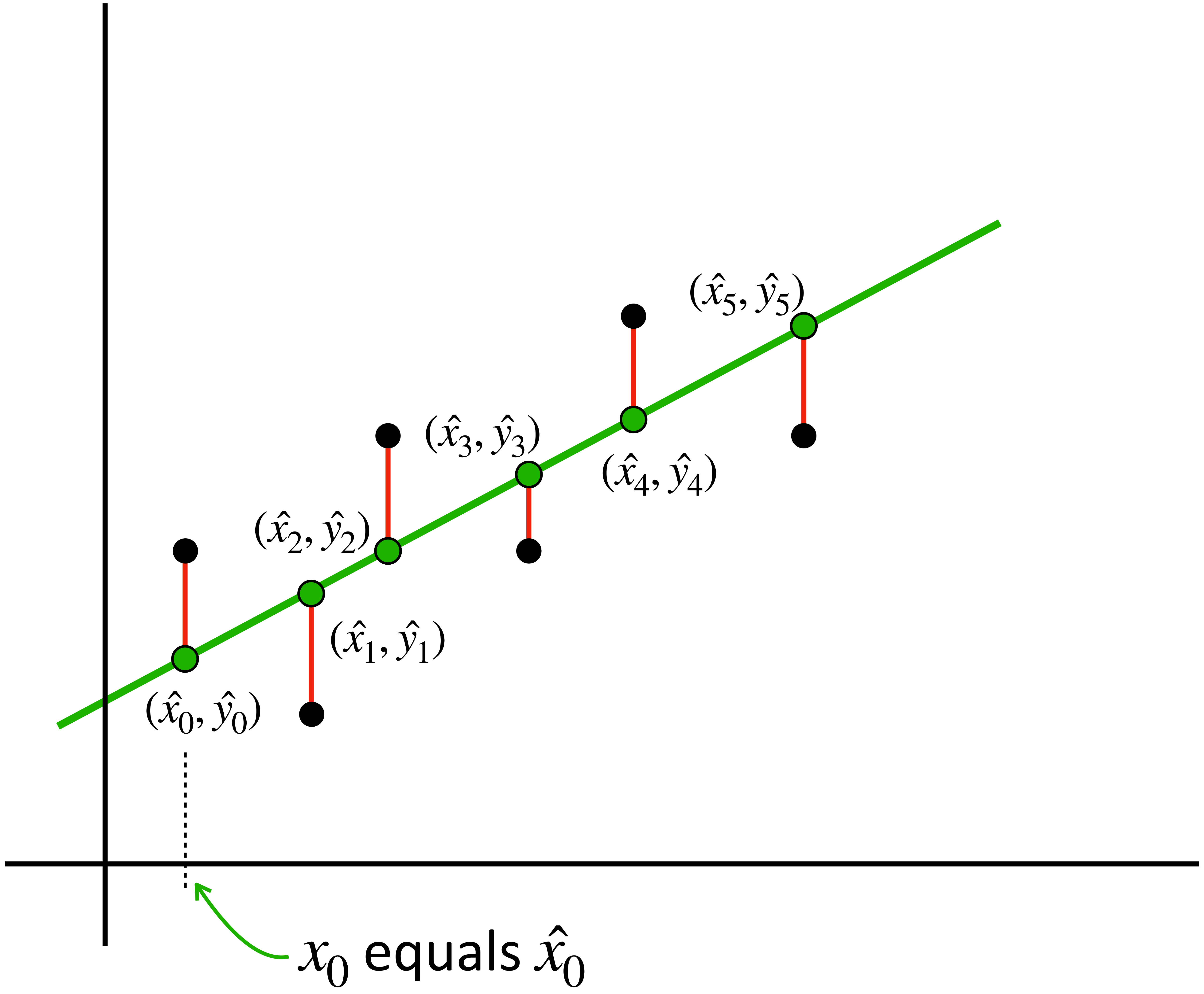
For every value of $i$, $\hat{x}_i$ equals $x_i$
Substitute $x_i$ for $\hat{x}_i$



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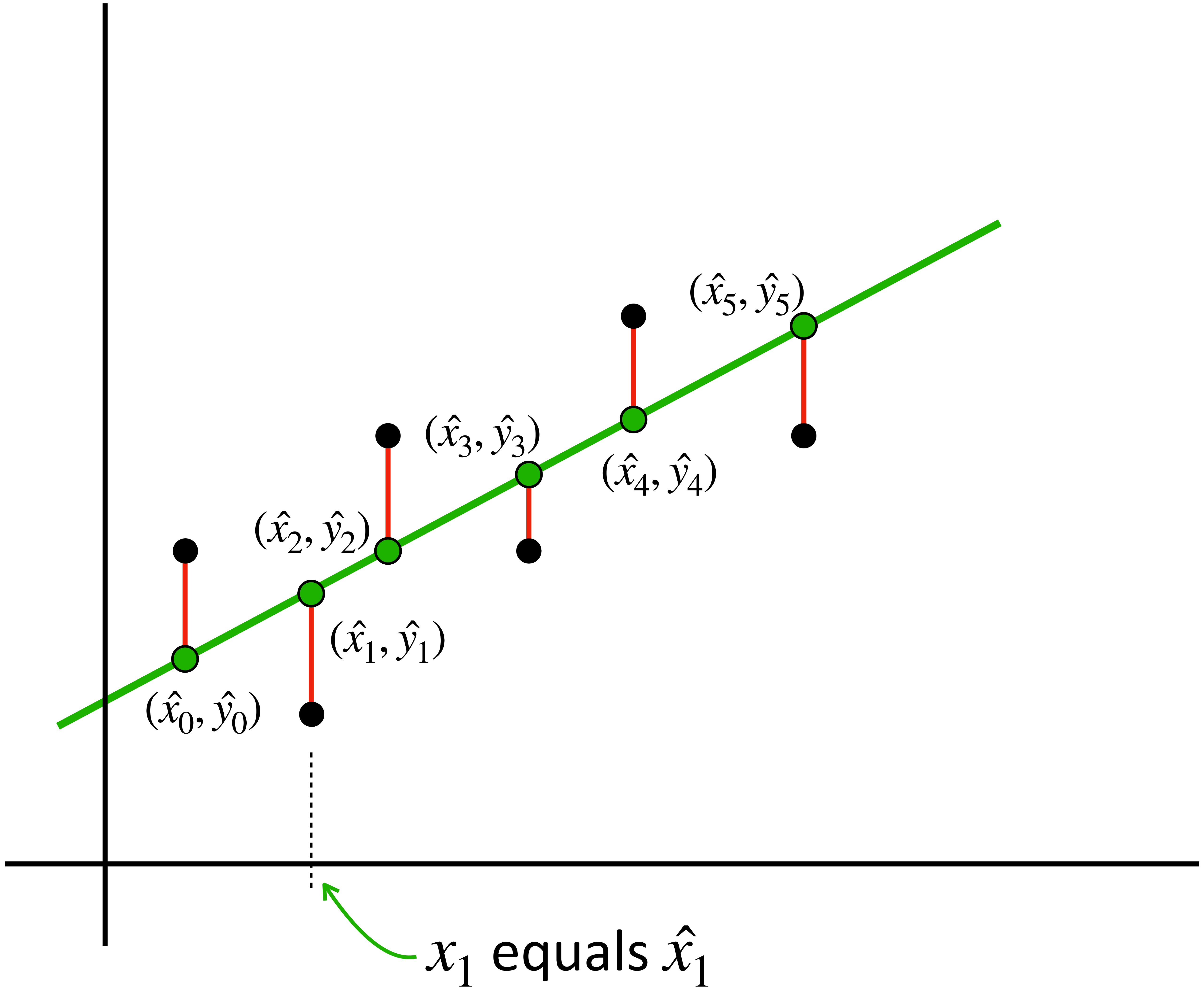


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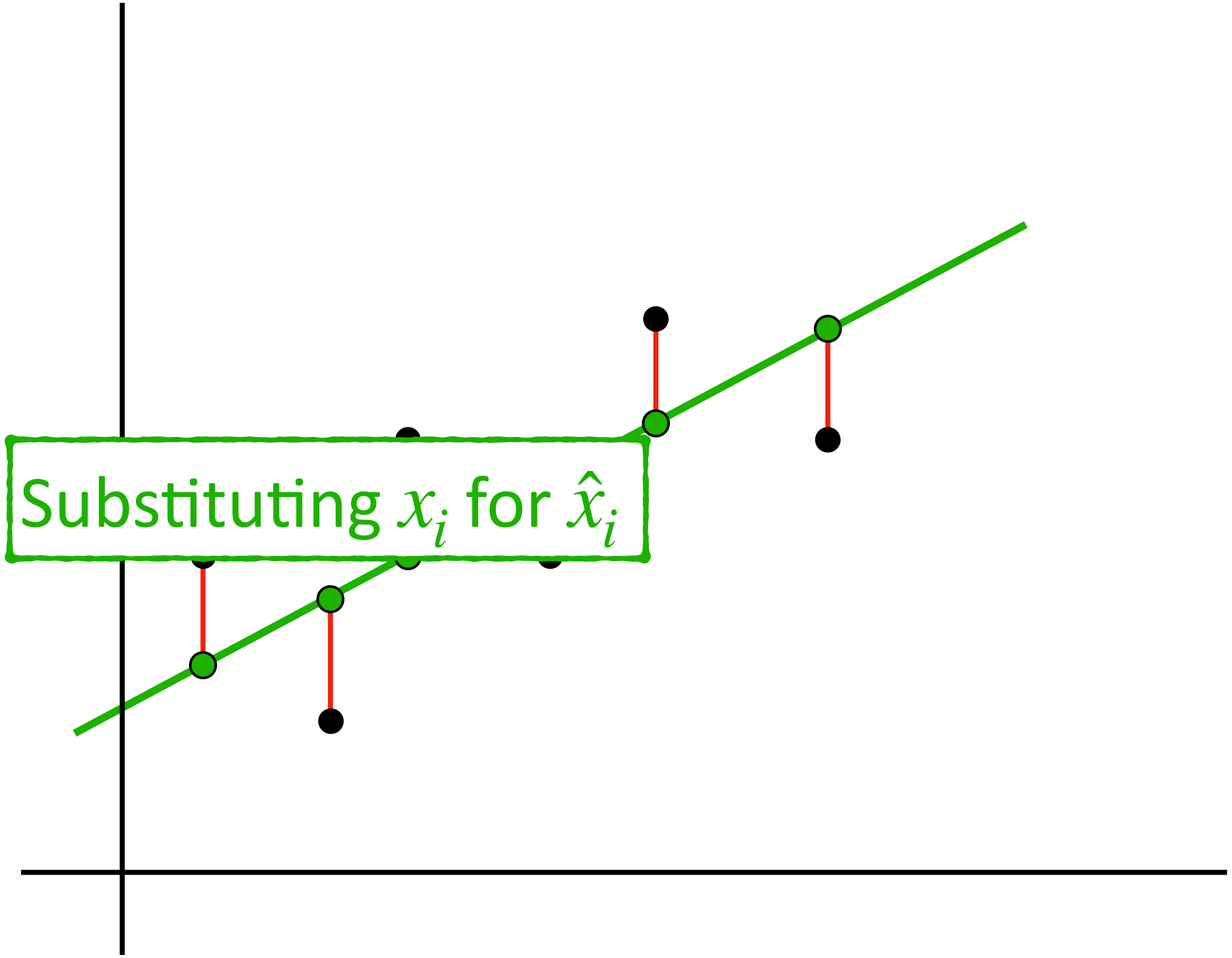

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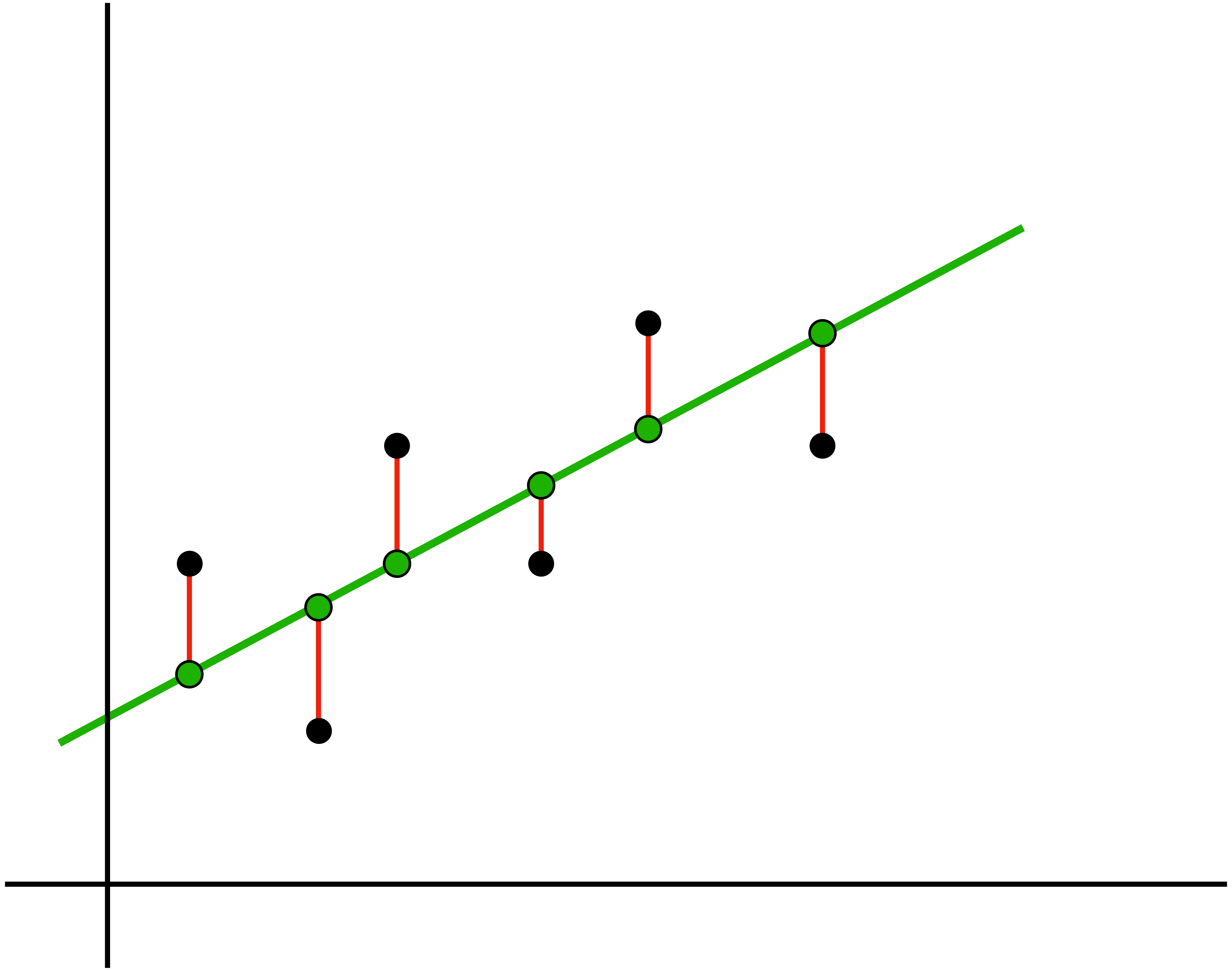

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**Solution:**

$$\beta_0 = \frac{\sum_{i=0}^{n} y_i - \beta_1 \sum_{i=0}^{n} x_i}{n}$$

$$\beta_1 = \frac{n \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i}{n \sum_{i=0}^{n} x_i^2 - \left( \sum_{i=0}^{n} x_i \right)^2}$$

Lets walk through the proof…

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$$\frac{\partial}{\partial\beta_0}\frac{1}{n}\sum_{i=0}^{n}(y_i - \beta_0 - \beta_1 x_i)^2 = 0 \quad \cdots\cdots\cdots\cdots\cdots \quad eq(1)$$

$$\frac{\partial}{\partial\beta_1}\frac{1}{n}\sum_{i=0}^{n}(y_i - \beta_0 - \beta_1 x_i)^2 = 0 \quad \cdots\cdots\cdots\cdots\cdots \quad eq(2)$$

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The Mean Squared Error (MSE) is minimized when the partial derivative is zero

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the MSE is minimized.

Solving equation 1 (take the partial derivative w.r.t $\beta_0$):

Chain Rule & Power Rule

See Tutorial on Derivatives

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$$\Rightarrow \frac{2}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) \frac{\partial}{\partial \beta_0} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0$$

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$$\Rightarrow \frac{2}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)(-1) = 0$$

Taking the partial derivative of $\sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)$

$$\frac{\partial}{\partial \beta_0} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) = (-1)$$

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Divide both sides by $-\frac{2}{n}$

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$$\Rightarrow \sum_{i=0}^{n} y_i - n\beta_0 - \beta_1 \sum_{i=0}^{n} x_i = 0$$

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Solving equation 1 (take the partial derivative w.r.t $\beta_0$):

$$\frac{\partial}{\partial \beta_0} \frac{1}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 = 0$$

$$\Rightarrow \frac{2}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) \frac{\partial}{\partial \beta_0} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0$$

Chain Rule & Power Rule

See Tutorial on Derivatives

$$\Rightarrow \frac{2}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)(-1) = 0$$

$$\Rightarrow \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\Rightarrow \sum_{i=0}^{n} y_i - n\beta_0 - \beta_1 \sum_{i=0}^{n} x_i = 0 \qquad \Rightarrow \beta_0 = \frac{\sum_{i=0}^{n} y_i - \beta_1 \sum_{i=0}^{n} x_i}{n}$$

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the MSE is minimized.

Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

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$$\Rightarrow \frac{2}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) \frac{\partial}{\partial \beta_1} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\Rightarrow \frac{2}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)(-x_i) = 0$$

Taking the partial derivative of $\sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)$

$$\frac{\partial}{\partial \beta_1} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) = (-x_i)$$

See Tutorial on Derivatives

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Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

$$\frac{\partial}{\partial \beta_1} \frac{1}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 = 0$$

$$\Rightarrow \frac{2}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) \frac{\partial}{\partial \beta_1} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\Rightarrow \frac{2}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)(-x_i) = 0$$

$$\Rightarrow \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)(x_i) = 0$$

Divide both sides by $-\frac{2}{n}$

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the MSE is minimized.

Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

$$\frac{\partial}{\partial \beta_1} \frac{1}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)^2 = 0$$

$$\Rightarrow \frac{2}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) \frac{\partial}{\partial \beta_1} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\Rightarrow \frac{2}{n} \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)(-x_i) = 0$$

$$\Rightarrow \sum_{i=0}^{n} (y_i - \beta_0 - \beta_1 x_i)(x_i) = 0$$

$$\Rightarrow \sum_{i=0}^{n} x_i y_i - \beta_0 \sum_{i=0}^{n} x_i - \beta_1 \sum_{i=0}^{n} x_i^2 = 0$$

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the MSE is minimized.

Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

$$\Rightarrow \sum_{i=0}^{n} x_i y_i - \beta_0 \sum_{i=0}^{n} x_i - \beta_1 \sum_{i=0}^{n} x_i^2 = 0$$

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Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

$$\Rightarrow \sum_{i=0}^{n} x_i y_i - \beta_0 \sum_{i=0}^{n} x_i - \beta_1 \sum_{i=0}^{n} x_i^2 = 0$$

Substitute $\beta_0 = \dfrac{\sum_{i=0}^{n} y_i - \beta_1 \sum_{i=0}^{n} x_i}{n}$

$$\Rightarrow \sum_{i=0}^{n} x_i y_i - \beta_1 \sum_{i=0}^{n} x_i^2 - \left( \frac{\sum_{i=0}^{n} y_i - \beta_1 \sum_{i=0}^{n} x_i}{n} \right) \sum_{i=0}^{n} x_i = 0$$

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$$\Rightarrow n \sum_{i=0}^{n} x_i y_i - \beta_1 n \sum_{i=0}^{n} x_i^2 - \left( \sum_{i=0}^{n} y_i \sum_{i=0}^{n} x_i - \beta_1 \left( \sum_{i=0}^{n} x_i \right)^2 \right) = 0$$

Divide both sides by $n$ and simplify

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the MSE is minimized.

Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

$$\Rightarrow \sum_{i=0}^{n} x_i y_i - \beta_0 \sum_{i=0}^{n} x_i - \beta_1 \sum_{i=0}^{n} x_i^2 = 0$$

$$\Rightarrow \sum_{i=0}^{n} x_i y_i - \beta_1 \sum_{i=0}^{n} x_i^2 - \left( \frac{\sum_{i=0}^{n} y_i - \beta_1 \sum_{i=0}^{n} x_i}{n} \right) \sum_{i=0}^{n} x_i = 0$$

$$\Rightarrow n \sum_{i=0}^{n} x_i y_i - \beta_1 n \sum_{i=0}^{n} x_i^2 - \left( \sum_{i=0}^{n} y_i \sum_{i=0}^{n} x_i - \beta_1 \left( \sum_{i=0}^{n} x_i \right)^2 \right) = 0$$

$$\Rightarrow n \sum_{i=0}^{n} x_i y_i - \beta_1 n \sum_{i=0}^{n} x_i^2 - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i + \beta_1 \left( \sum_{i=0}^{n} x_i \right)^2 = 0$$

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the MSE is minimized.

Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

$$\Rightarrow n\sum_{i=0}^{n} x_i y_i - \beta_1 n \sum_{i=0}^{n} x_i^2 - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i + \beta_1 \left( \sum_{i=0}^{n} x_i \right)^2 = 0$$

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$$\Rightarrow \beta_1 \left(\sum_{i=0}^{n} x_i\right)^2 - \beta_1 n \sum_{i=0}^{n} x_i^2 = \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i - n\sum_{i=0}^{n} x_i y_i$$

Add $\sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i - n\sum_{i=0}^{n} x_i y_i$ to both sides

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the MSE is minimized.

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$$\Rightarrow n\sum_{i=0}^{n} x_i y_i - \beta_1 n \sum_{i=0}^{n} x_i^2 - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i + \beta_1 \left(\sum_{i=0}^{n} x_i\right)^2 = 0$$

$$\Rightarrow \beta_1 \left(\sum_{i=0}^{n} x_i\right)^2 - \beta_1 n \sum_{i=0}^{n} x_i^2 = \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i - n\sum_{i=0}^{n} x_i y_i$$

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$$\Rightarrow \beta_1 \left(\sum_{i=0}^{n} x_i\right)^2 - \beta_1 n \sum_{i=0}^{n} x_i^2 = \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i - n\sum_{i=0}^{n} x_i y_i$$

$$\Rightarrow \beta_1 n \sum_{i=0}^{n} x_i^2 - \beta_1 \left(\sum_{i=0}^{n} x_i\right)^2 = n\sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i$$

Multiply both sides by $-1$

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the MSE is minimized.

Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

$$\Rightarrow n\sum_{i=0}^{n} x_i y_i - \beta_1 n \sum_{i=0}^{n} x_i^2 - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i + \beta_1 \left(\sum_{i=0}^{n} x_i\right)^2 = 0$$

$$\Rightarrow \beta_1 \left(\sum_{i=0}^{n} x_i\right)^2 - \beta_1 n \sum_{i=0}^{n} x_i^2 = \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i - n\sum_{i=0}^{n} x_i y_i$$

$$\Rightarrow \beta_1 n \sum_{i=0}^{n} x_i^2 - \beta_1 \left(\sum_{i=0}^{n} x_i\right)^2 = n\sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i$$

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Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

$$\Rightarrow \beta_1 n \sum_{i=0}^{n} x_i^2 - \beta_1 \left( \sum_{i=0}^{n} x_i \right)^2 = n \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i$$

Factor out $\beta_1$

$$\Rightarrow \beta_1 \left( n \sum_{i=0}^{n} x_i^2 - \left( \sum_{i=0}^{n} x_i \right)^2 \right) = n \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i$$

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the MSE is minimized.

Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

$$\Rightarrow \beta_1 n \sum_{i=0}^{n} x_i^2 - \beta_1 \left( \sum_{i=0}^{n} x_i \right)^2 = n \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i$$

$$\Rightarrow \beta_1 \left[ n \sum_{i=0}^{n} x_i^2 - \left( \sum_{i=0}^{n} x_i \right)^2 \right] = n \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i$$

$$\Rightarrow \beta_1 = \frac{n \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i}{n \sum_{i=0}^{n} x_i^2 - \left( \sum_{i=0}^{n} x_i \right)^2}$$

Divide both sides by $n \sum_{i=0}^{n} x_i^2 - \left( \sum_{i=0}^{n} x_i \right)^2$

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the MSE is minimized.

Solving equation 2 (take the partial derivative w.r.t $\beta_1$:

$$\Rightarrow \beta_1 n \sum_{i=0}^{n} x_i^2 - \beta_1 \left( \sum_{i=0}^{n} x_i \right)^2 = n \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i$$

$$\Rightarrow \beta_1 \left( n \sum_{i=0}^{n} x_i^2 - \left( \sum_{i=0}^{n} x_i \right)^2 \right) = n \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i$$

$$\Rightarrow \beta_1 = \frac{n \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i}{n \sum_{i=0}^{n} x_i^2 - \left( \sum_{i=0}^{n} x_i \right)^2}$$

**Q.E.D**

**Least Squares Regression:** Find the values of $\beta_0$ and $\beta_1$ such that the Mean Squared Error (MSE) is minimized.

**Solution:**

$$\beta_0 = \frac{\sum_{i=0}^{n} y_i - \beta_1 \sum_{i=0}^{n} x_i}{n}$$

$$\beta_1 = \frac{n \sum_{i=0}^{n} x_i y_i - \sum_{i=0}^{n} x_i \sum_{i=0}^{n} y_i}{n \sum_{i=0}^{n} x_i^2 - \left( \sum_{i=0}^{n} x_i \right)^2}$$

# Related Tutorials & Textbooks

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A statistical technique of making predictions from data. The tutorial introduces a linear model in two dimensions and uses that model to predict the value of one dependent variable given one independent variable.

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Multiple regression extends the two dimensional linear model introduced in Simple Linear Regression to k + 1 dimensions with one dependent variable, k independent variables and k+1 parameters.

## Gradient Descent for Simple Linear Regression

An introduction to the Gradient Descent algorithm and a deep dive on how it can be used to optimize the two parameters $\beta_0$ and $\beta_1$ for Simple Linear Regression.

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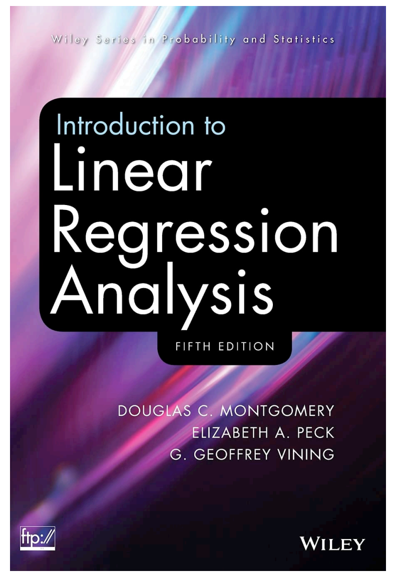


**Introduction to Linear Regression Analysis**

**by Douglas C. Montgomery, Elizabeth A. Peck, G. Geoffrey Vining**

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