Differential Calculus

Fundamentals

Rahul Singh
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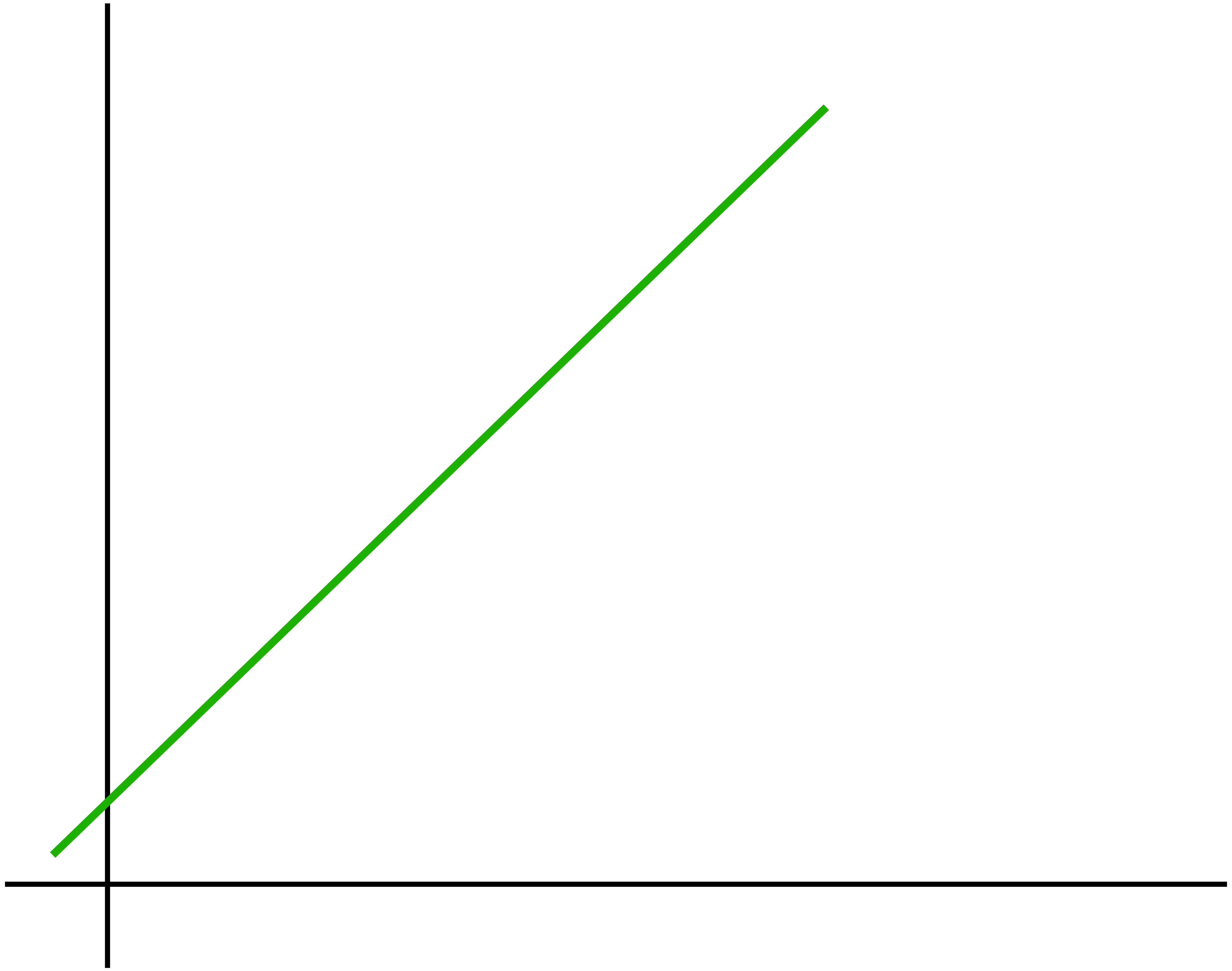
Equation for a Straight Line

$$y = mx + c$$

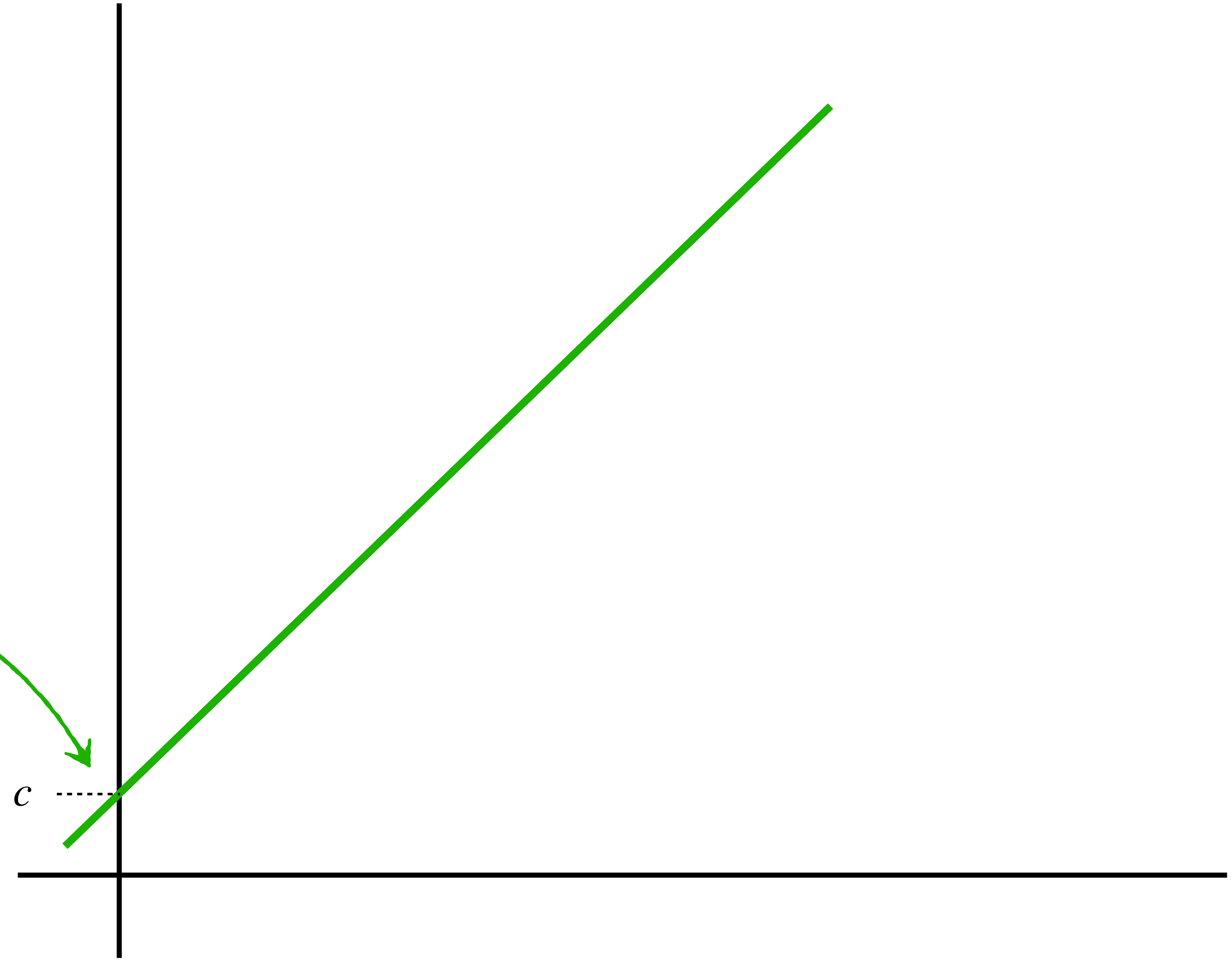


Equation for a Straight Line

$$y = mx + c$$

c Is the Y intercept

Y intercept: The point the line intersects the Y axis



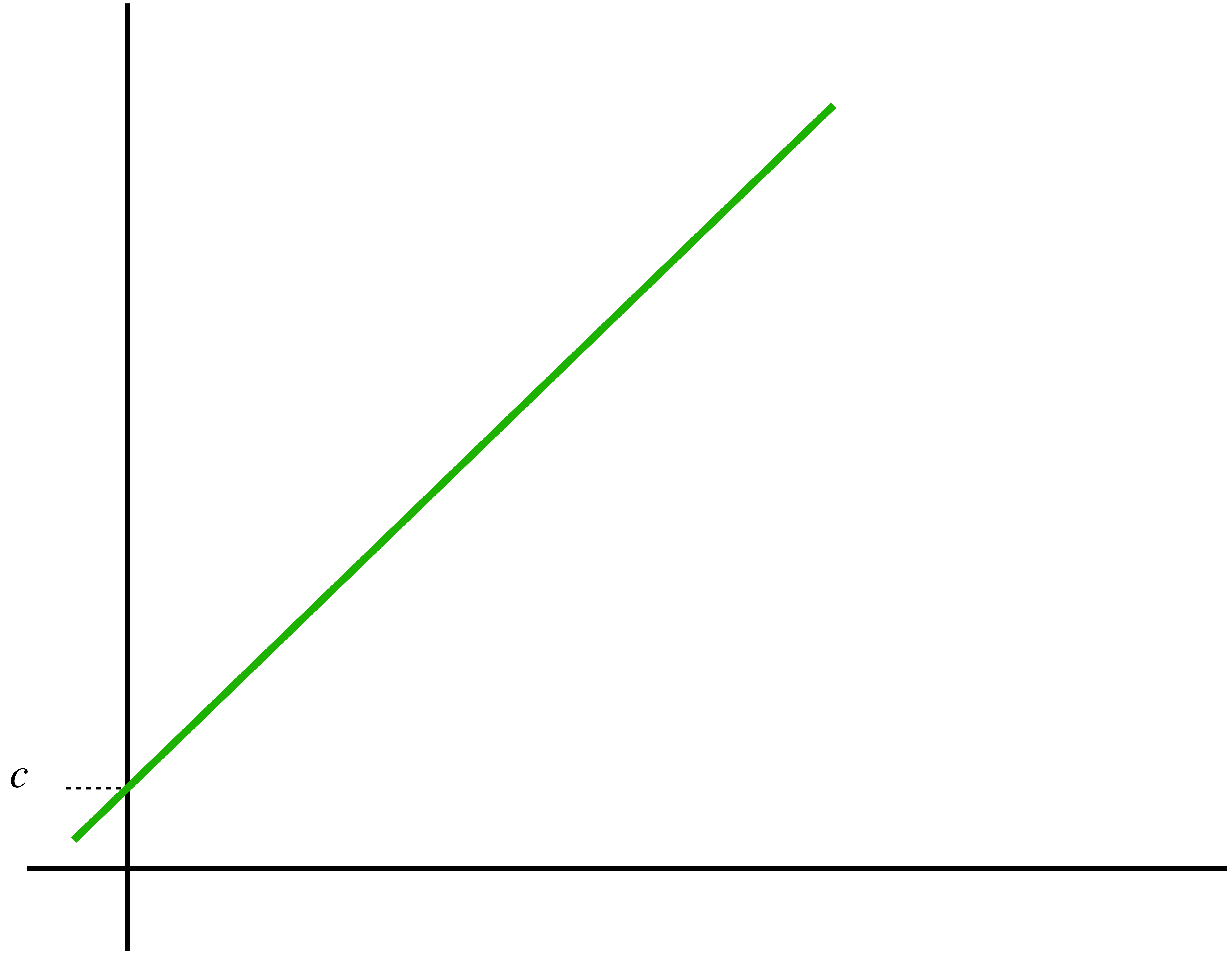
Equation for a Straight Line

$$y = mx + c$$

c Is the y intercept

m Is the slope of the line

What is the slope of the line?



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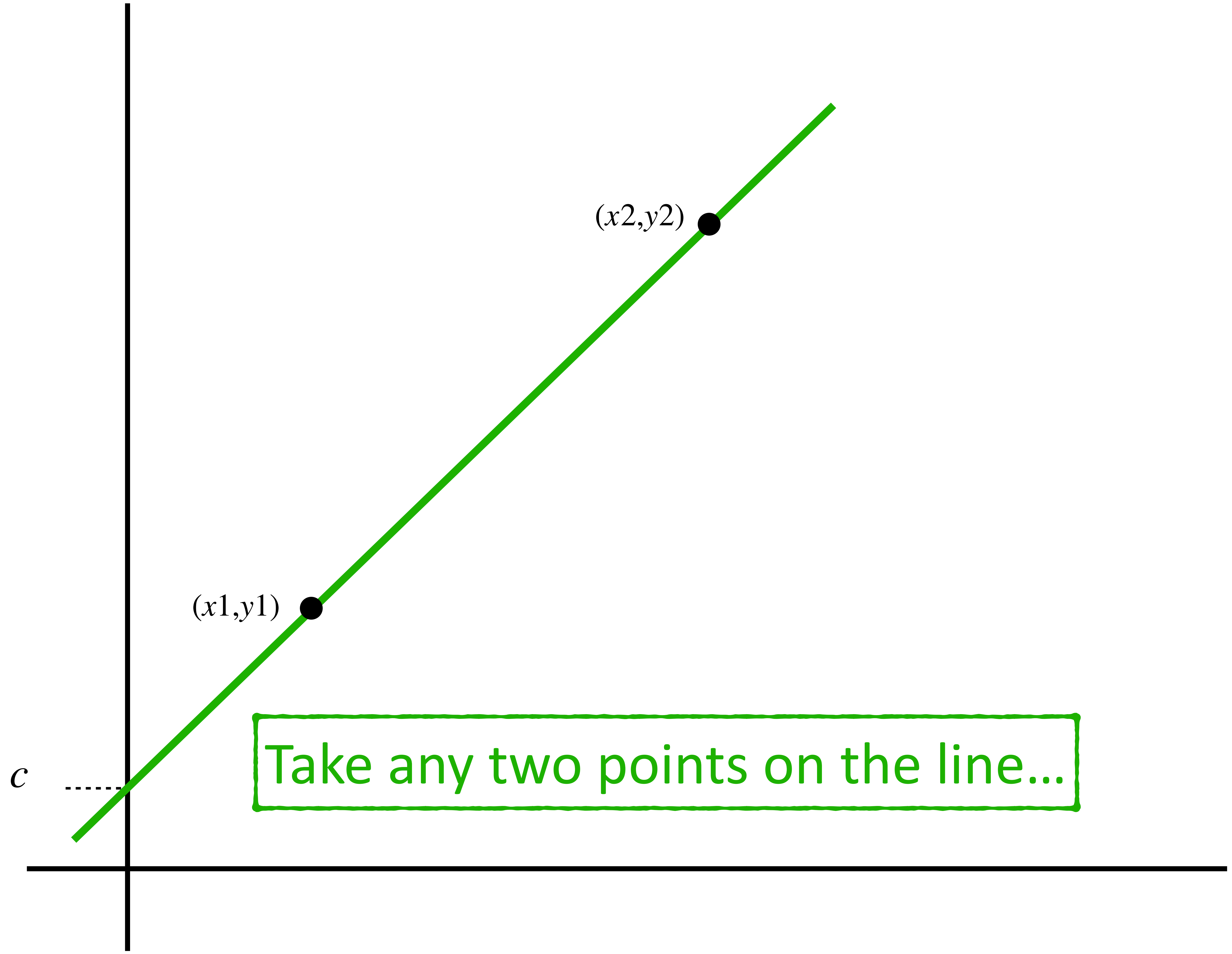

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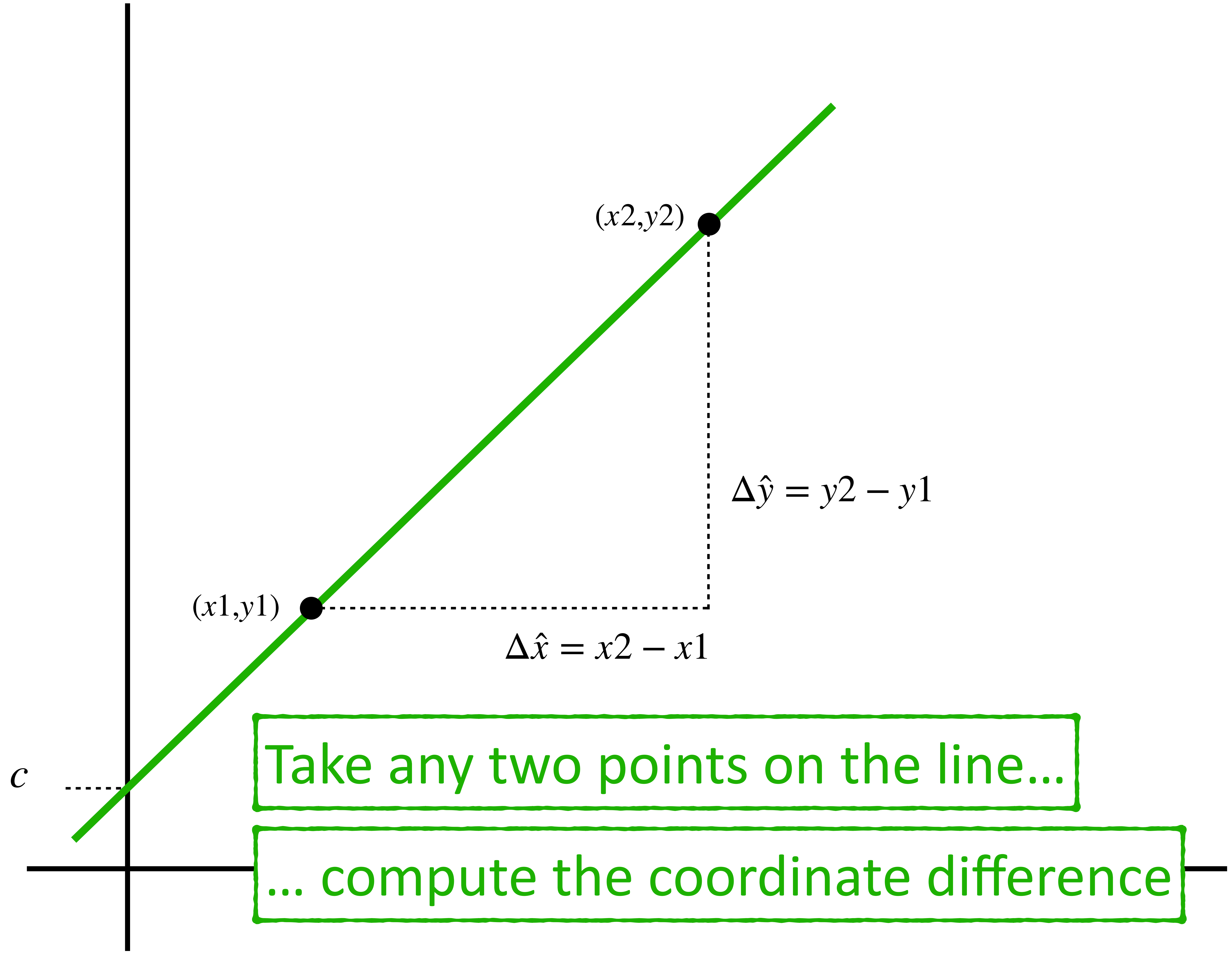

Take any two points on the line…

… compute the coordinate difference

Equation for a Straight Line

$$y = mx + c$$

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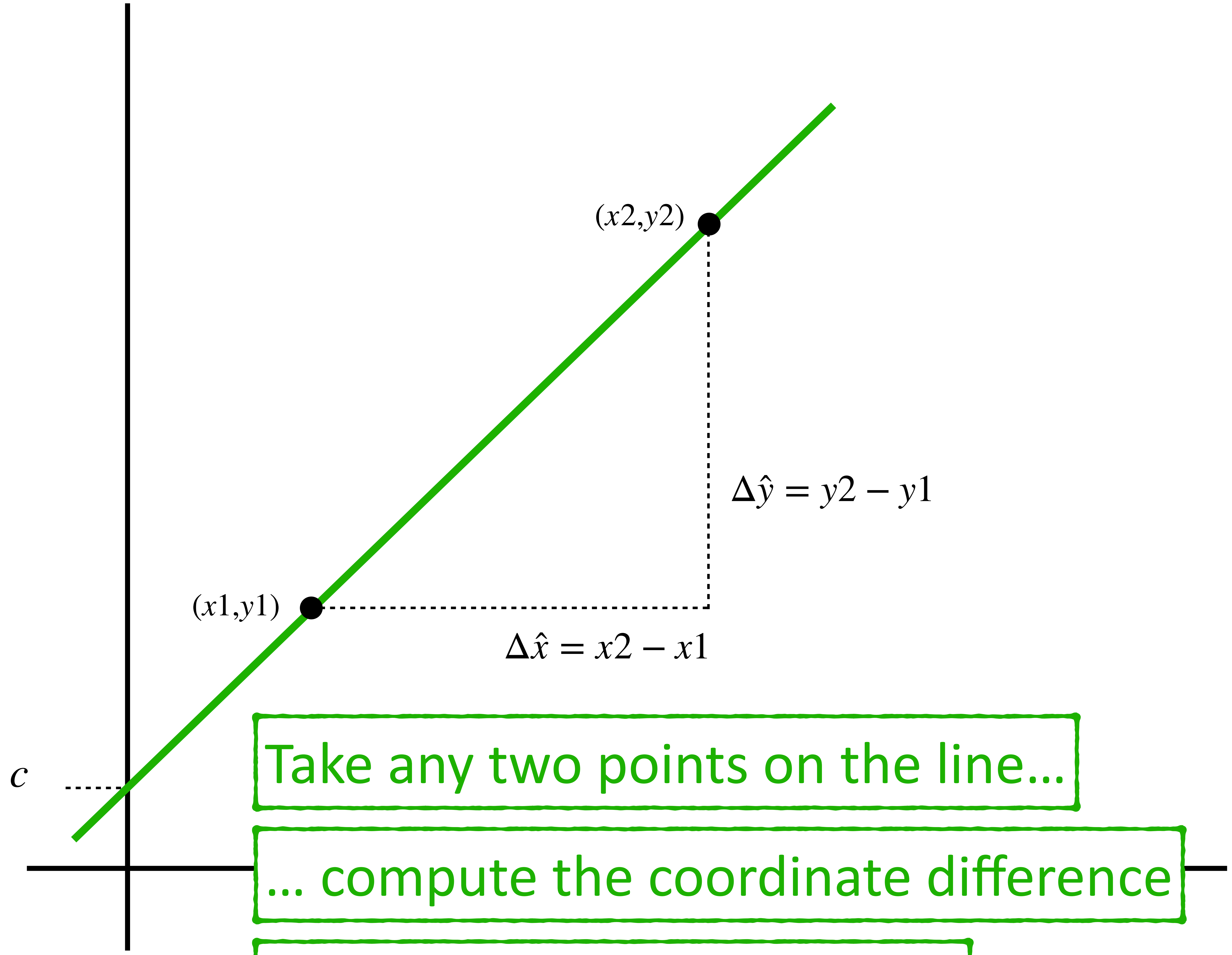
m Is the slope of the line

$$m = \frac{\Delta \hat{y}}{\Delta \hat{x}} = \frac{\hat{y}_2 - \hat{y}_1}{\hat{x}_2 - \hat{x}_1}$$

Slope of the line is the change in the y coordinate w.r.t to the change in the x coordinate



Take any two points on the line…

… compute the coordinate difference

Divide them to get the slope

Equation for a Straight Line

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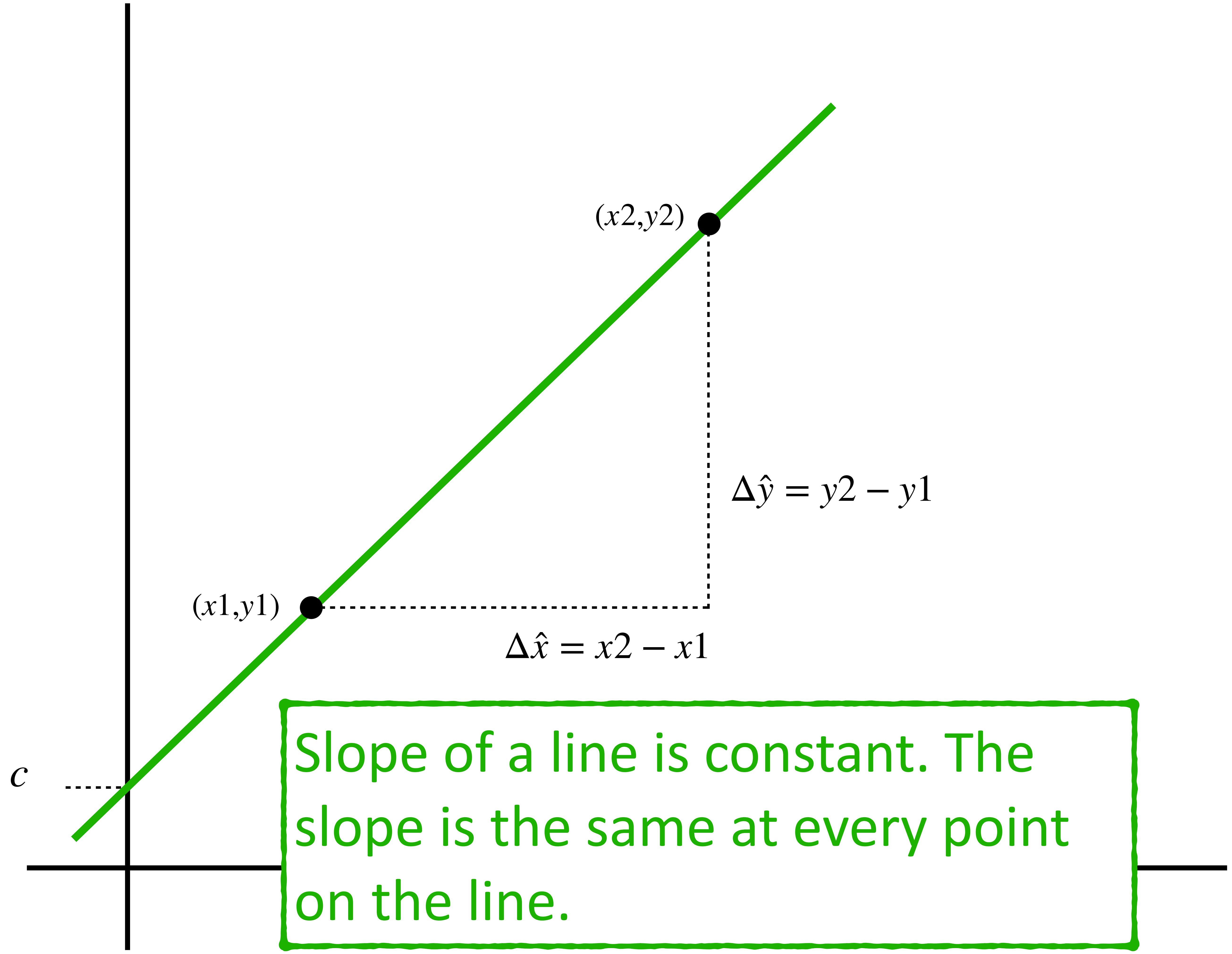
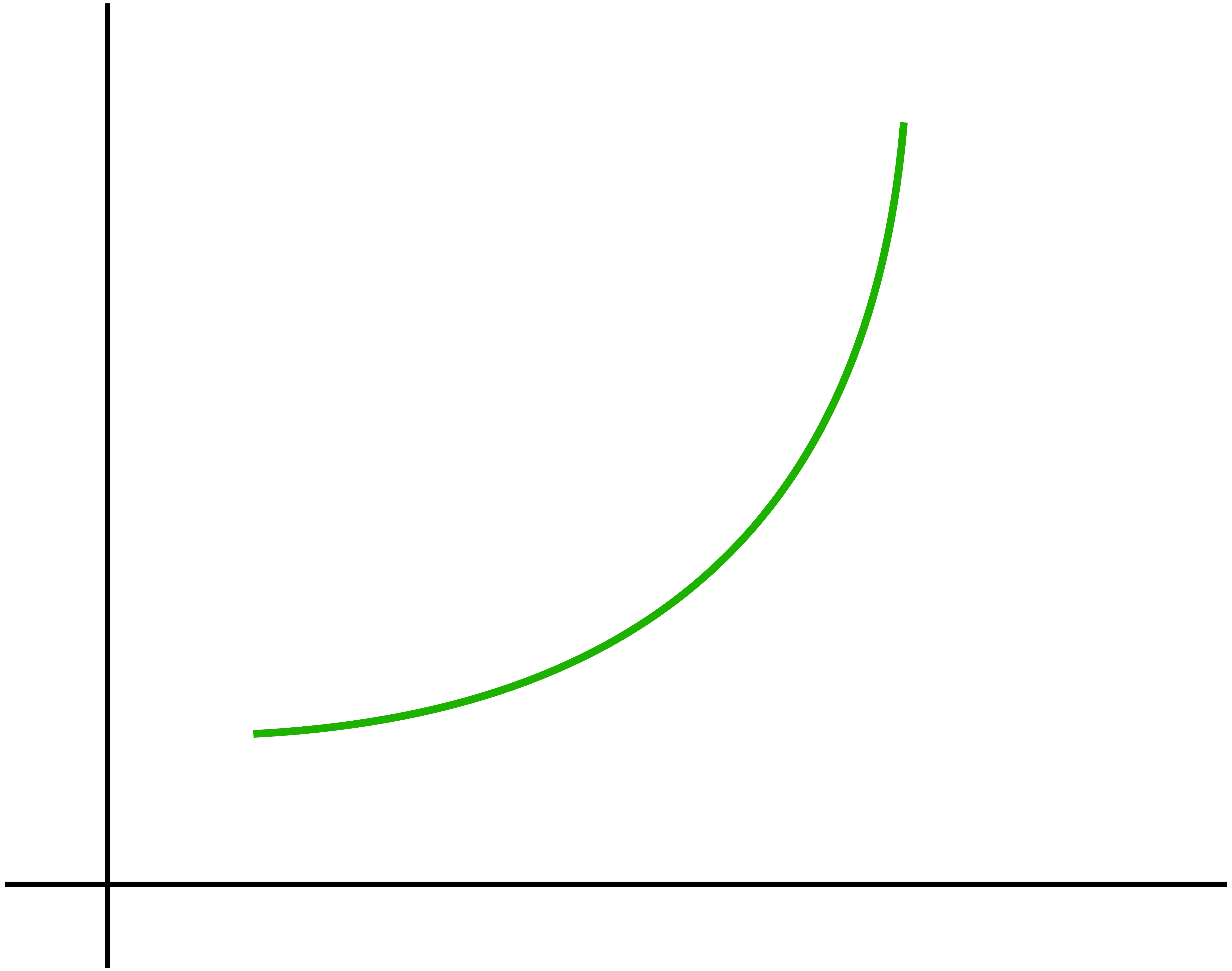


Slope of a line is constant. The slope is the same at every point on the line.

Slope of a Curve

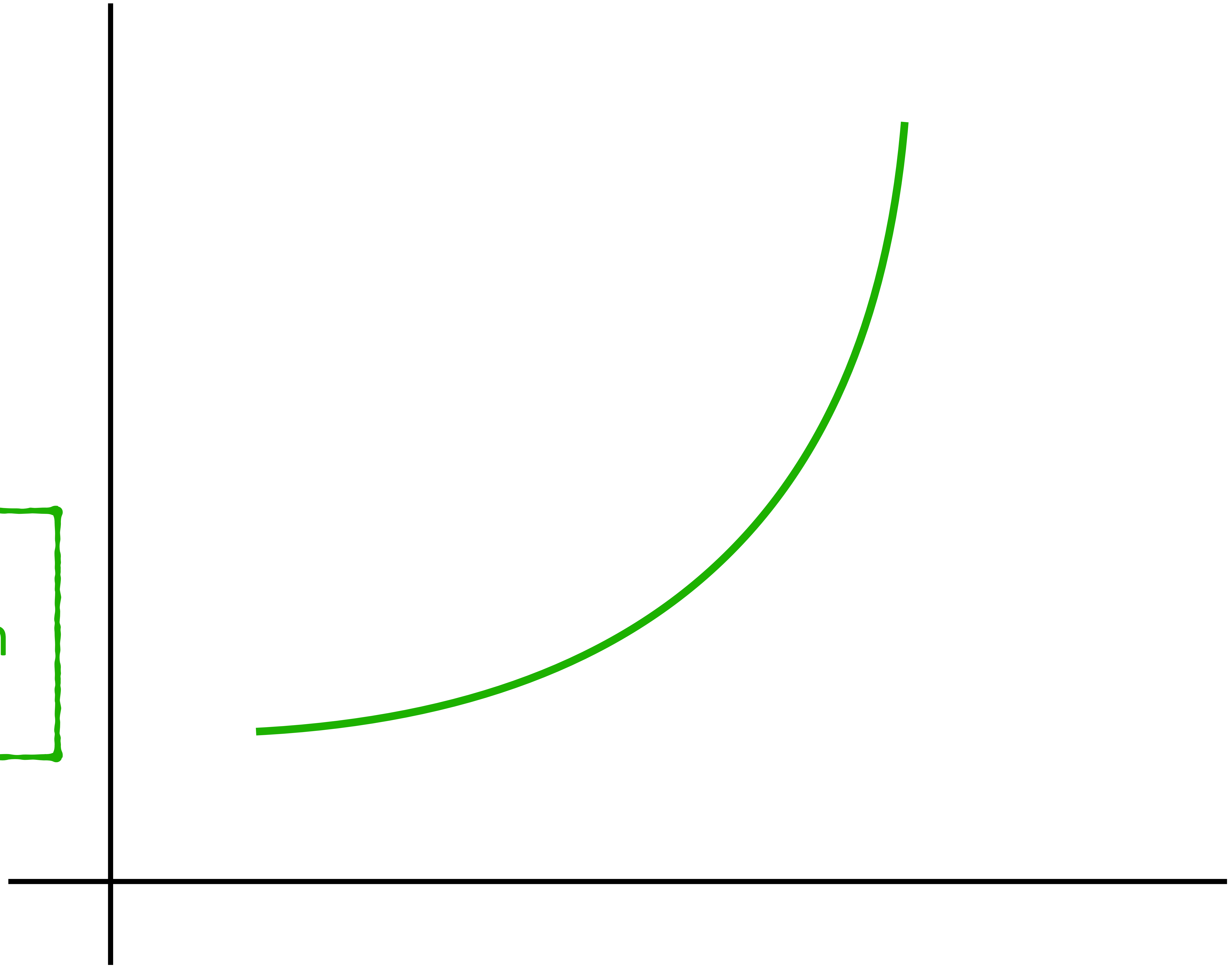
What is the slope of a curve?



Slope of a Curve

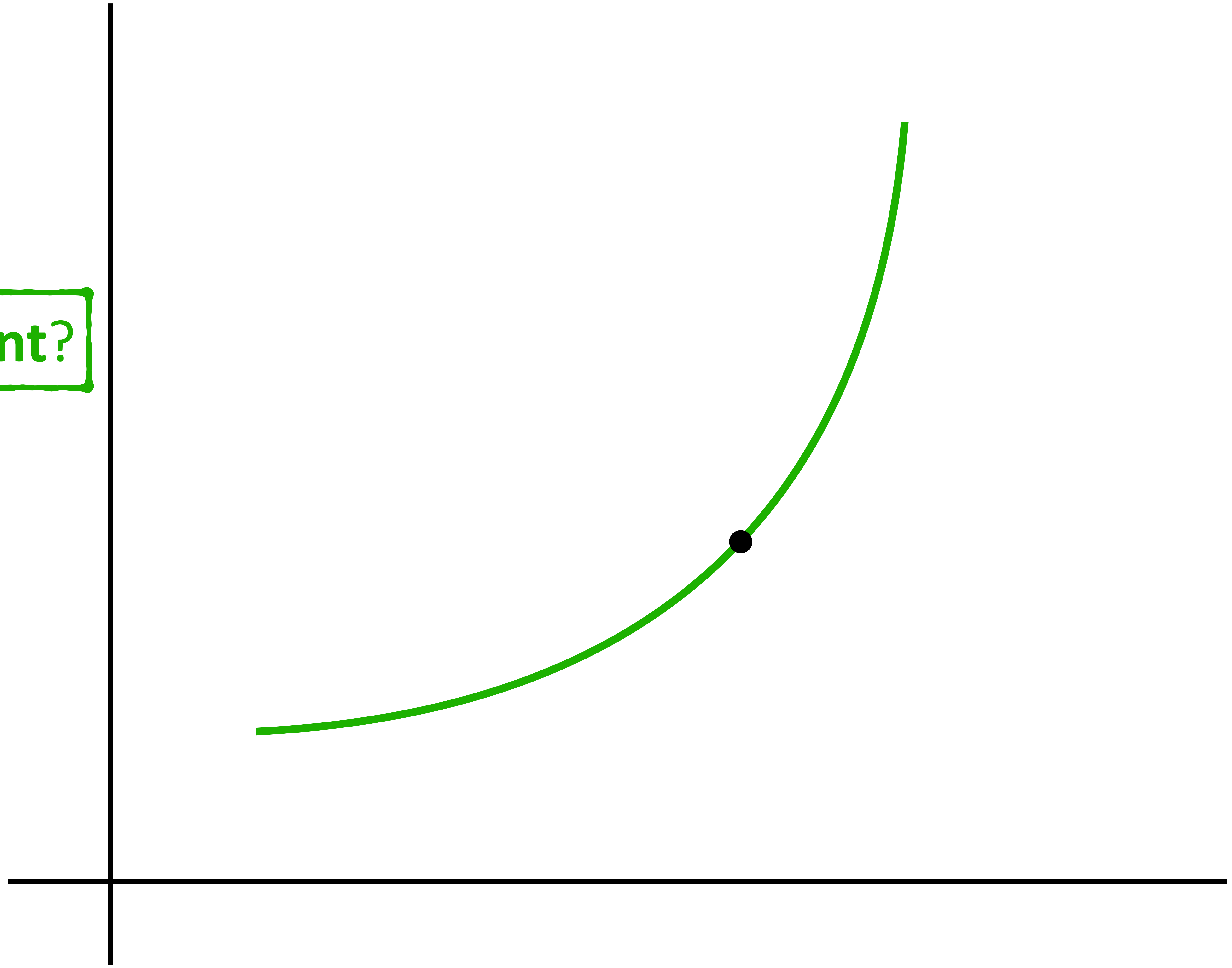
What is the slope of a curve?

Slope of a curve is not constant.
Slope of a curve is different at every point on the curve



Slope of a Curve

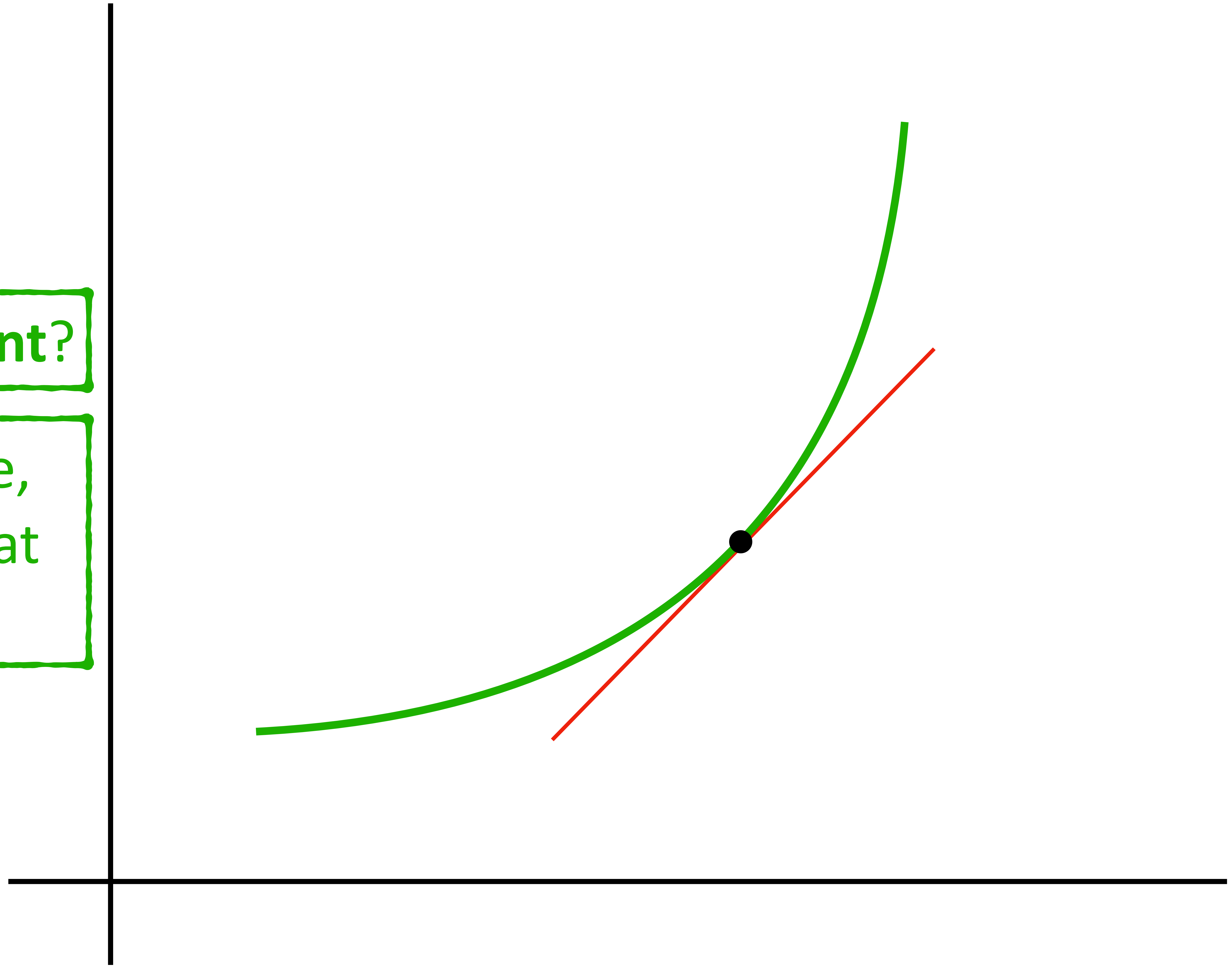
What is the slope of a curve **at a specific point**?



Slope of a Curve

What is the slope of a curve **at a specific point**?

Slope of a curve at a specific point on a curve, is the slope of the tangent to the curve at that point



Slope of a Curve

Problem Statement: How do you calculate the slope of the tangent at a given point

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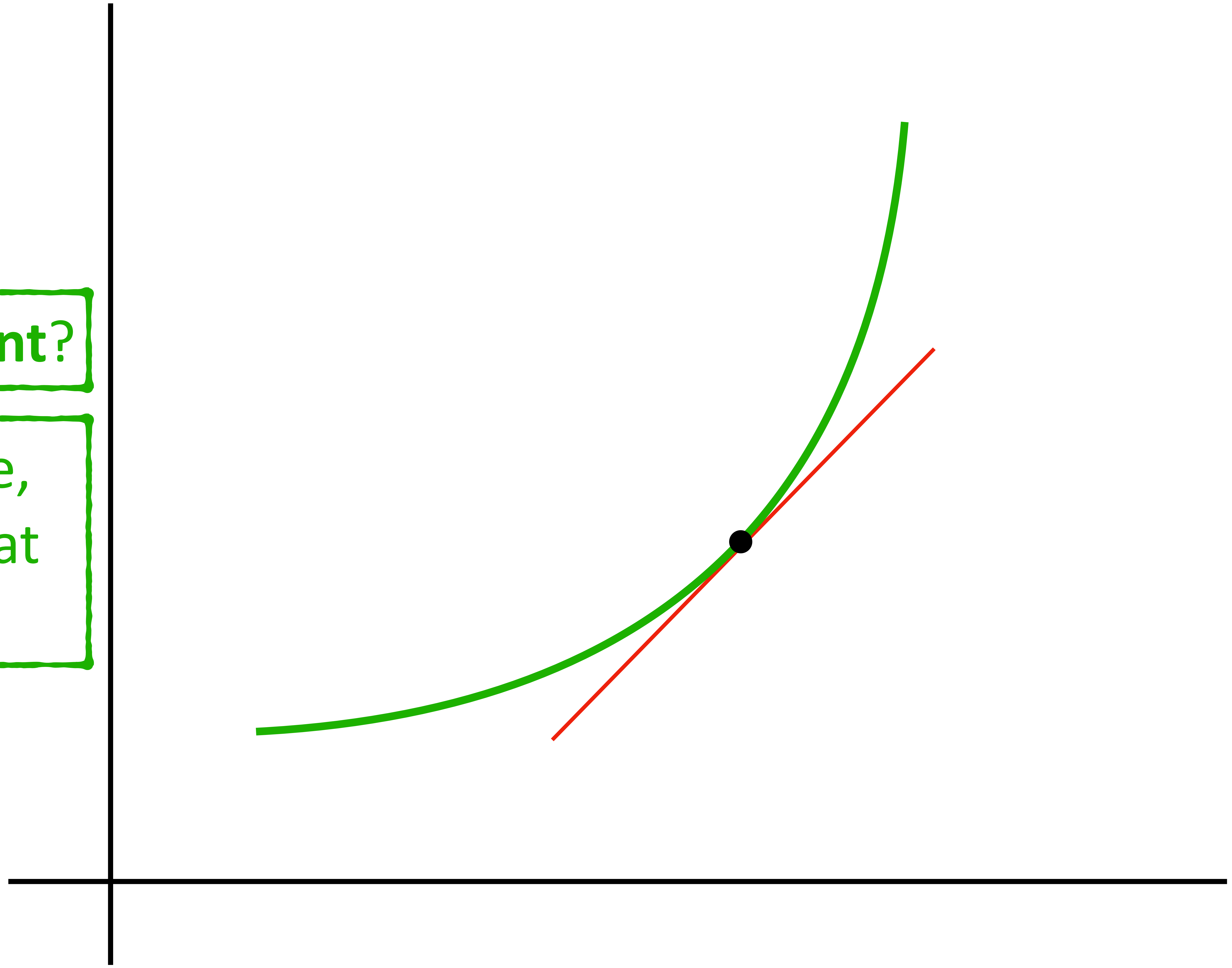

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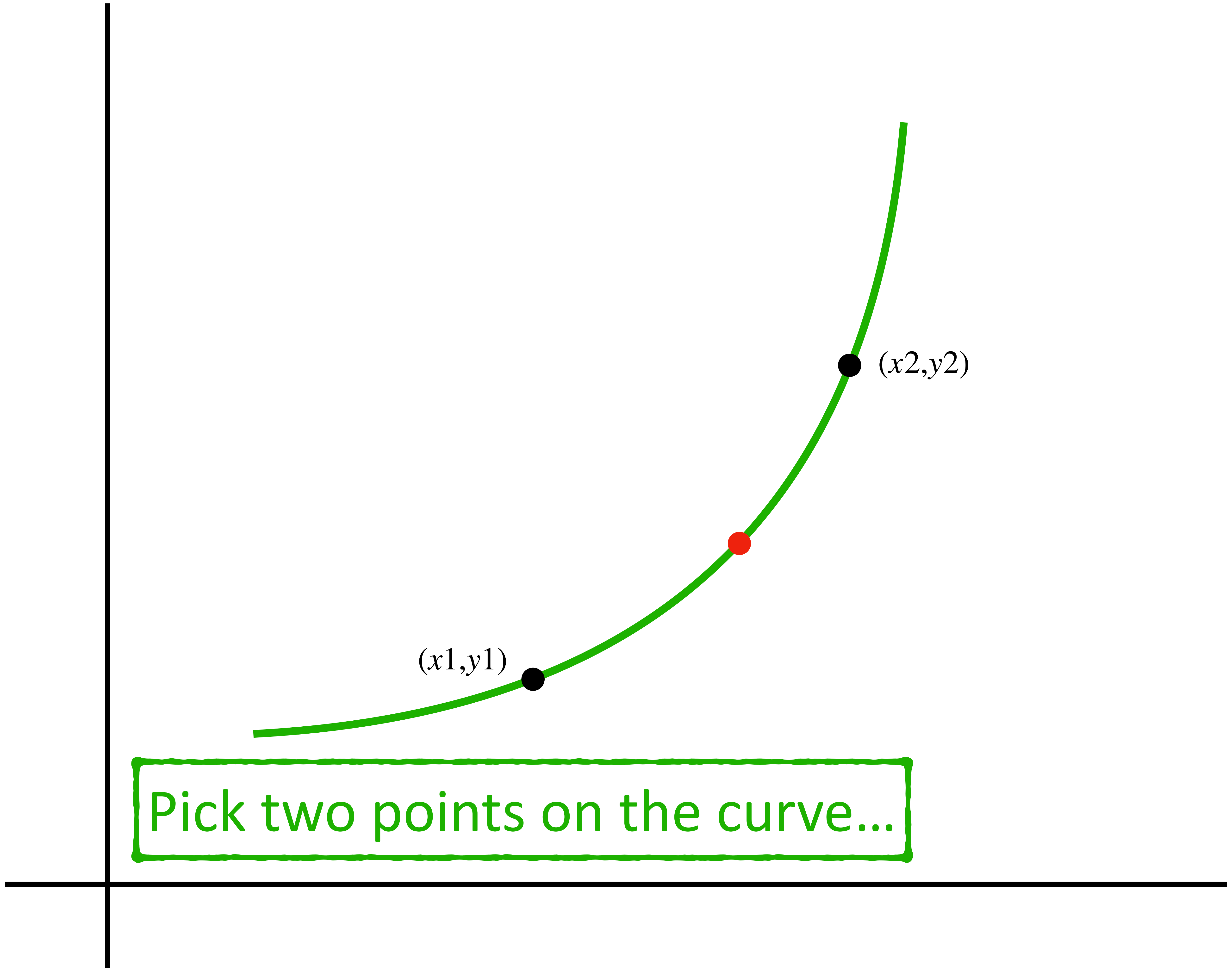

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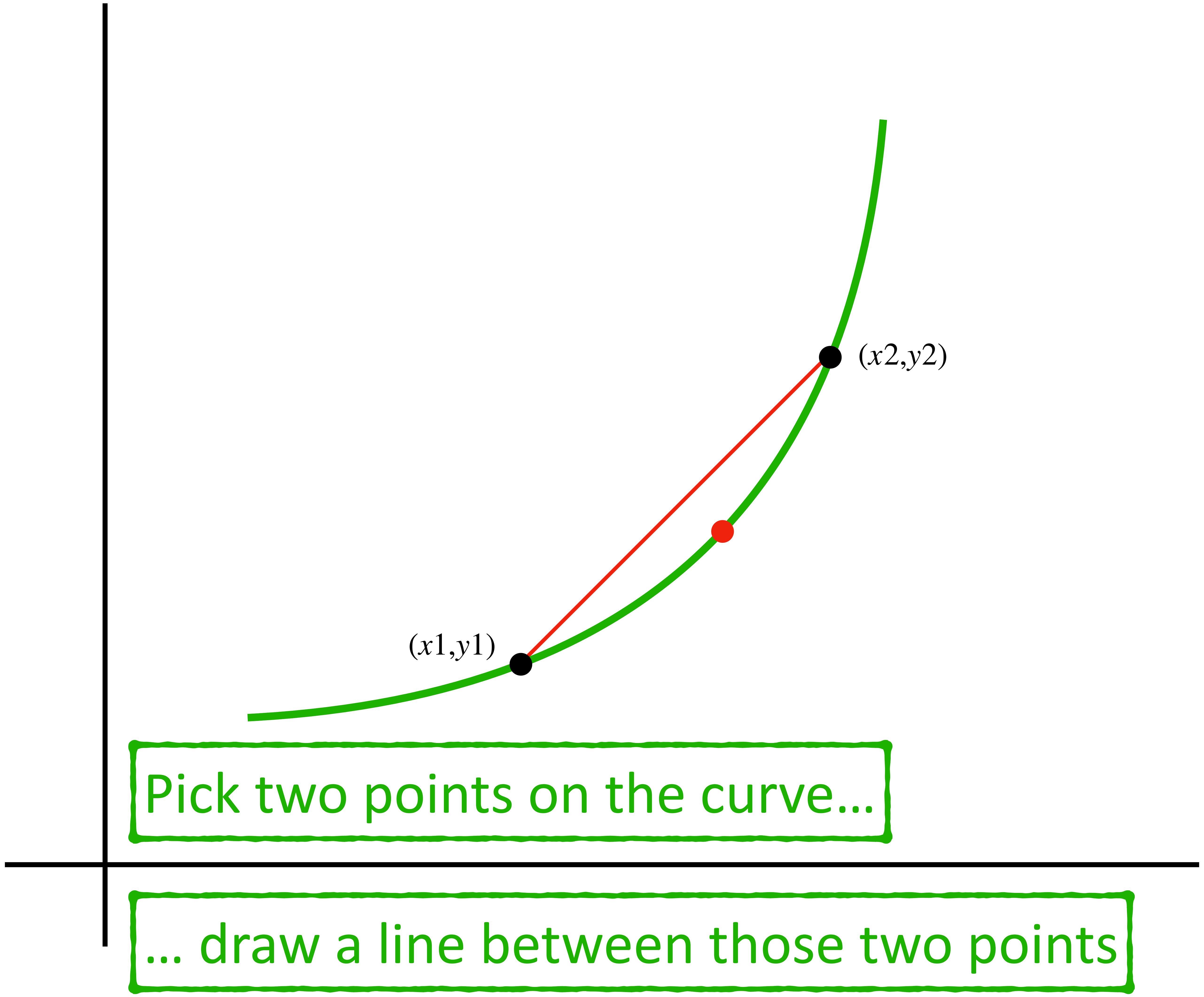

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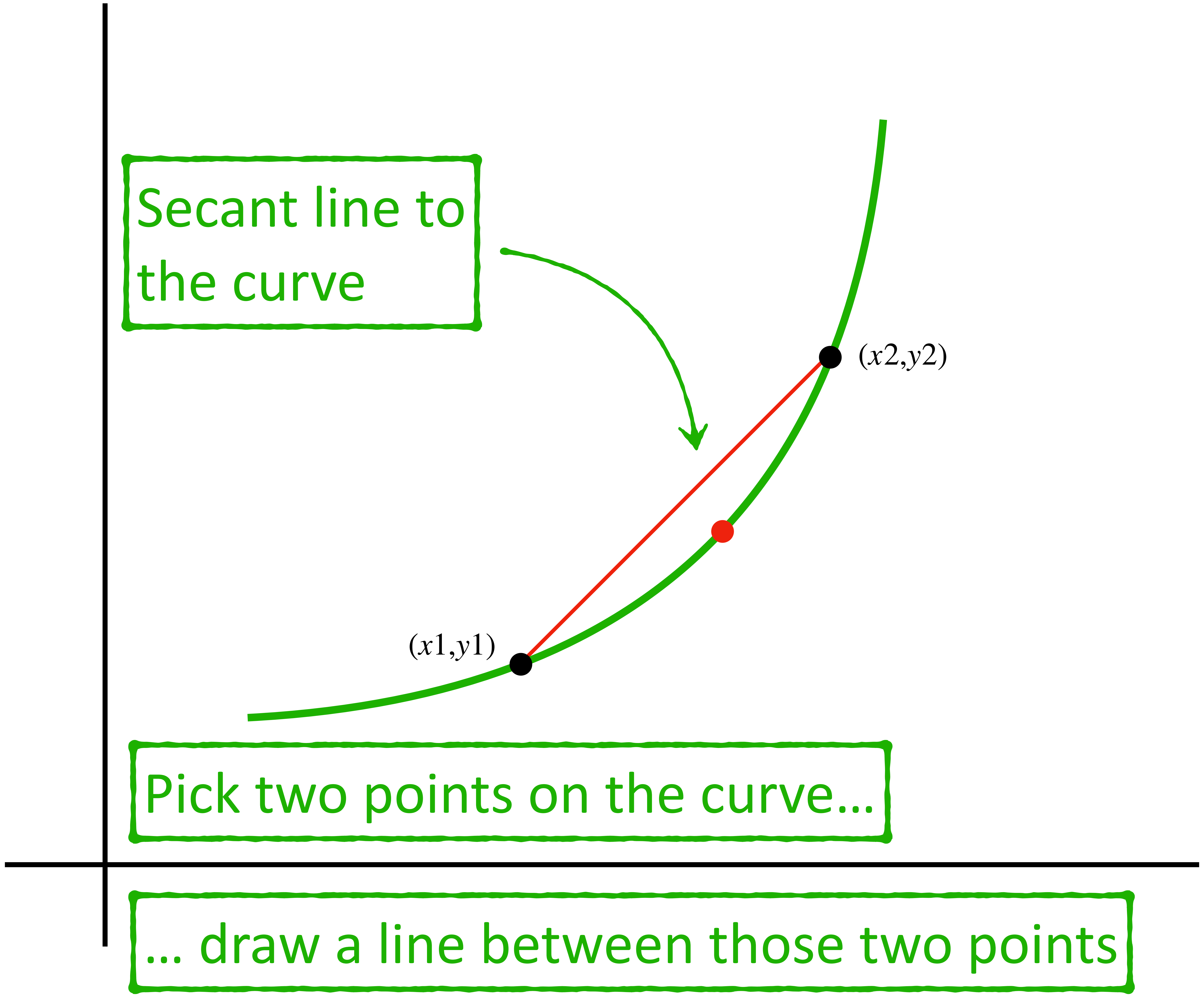

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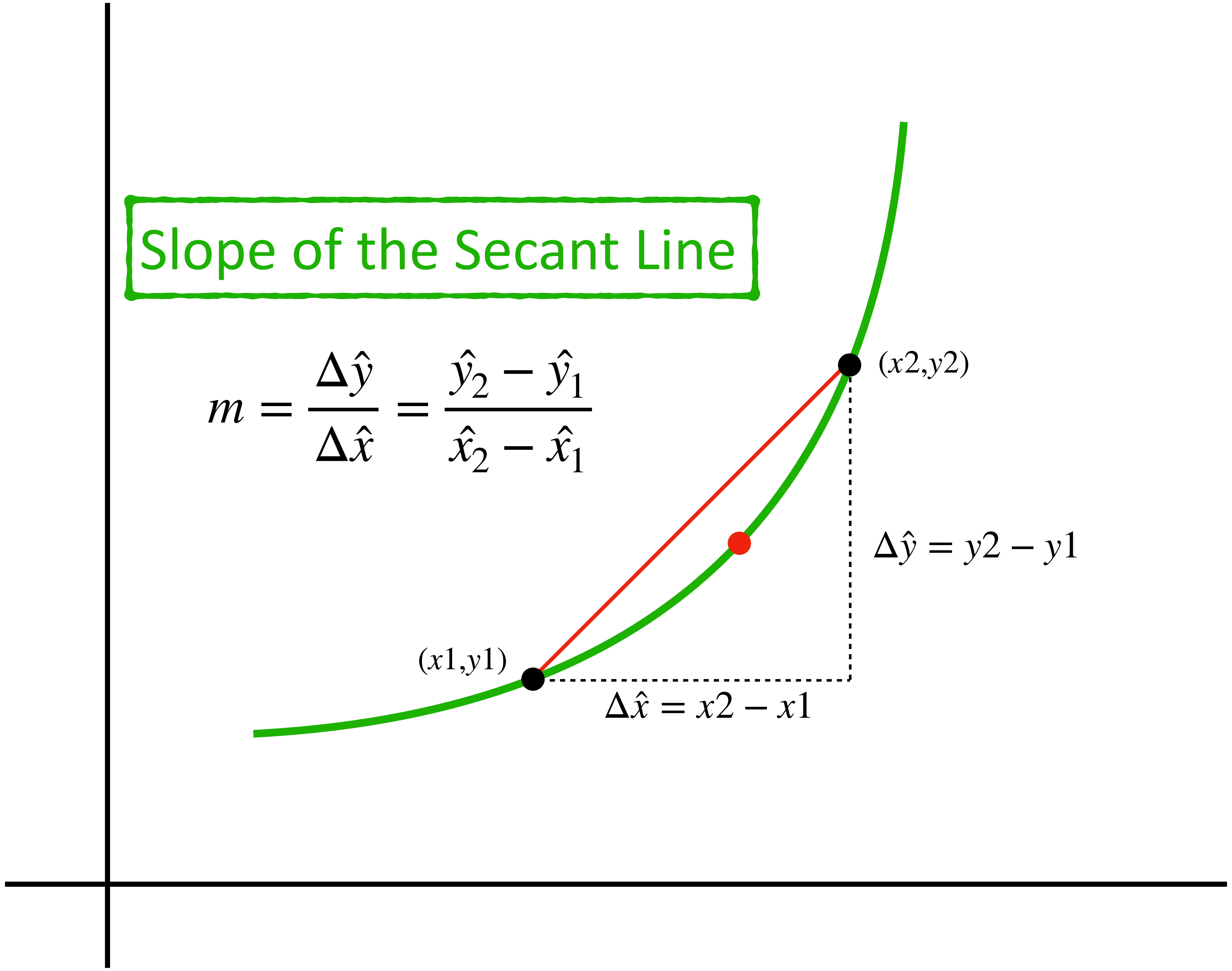

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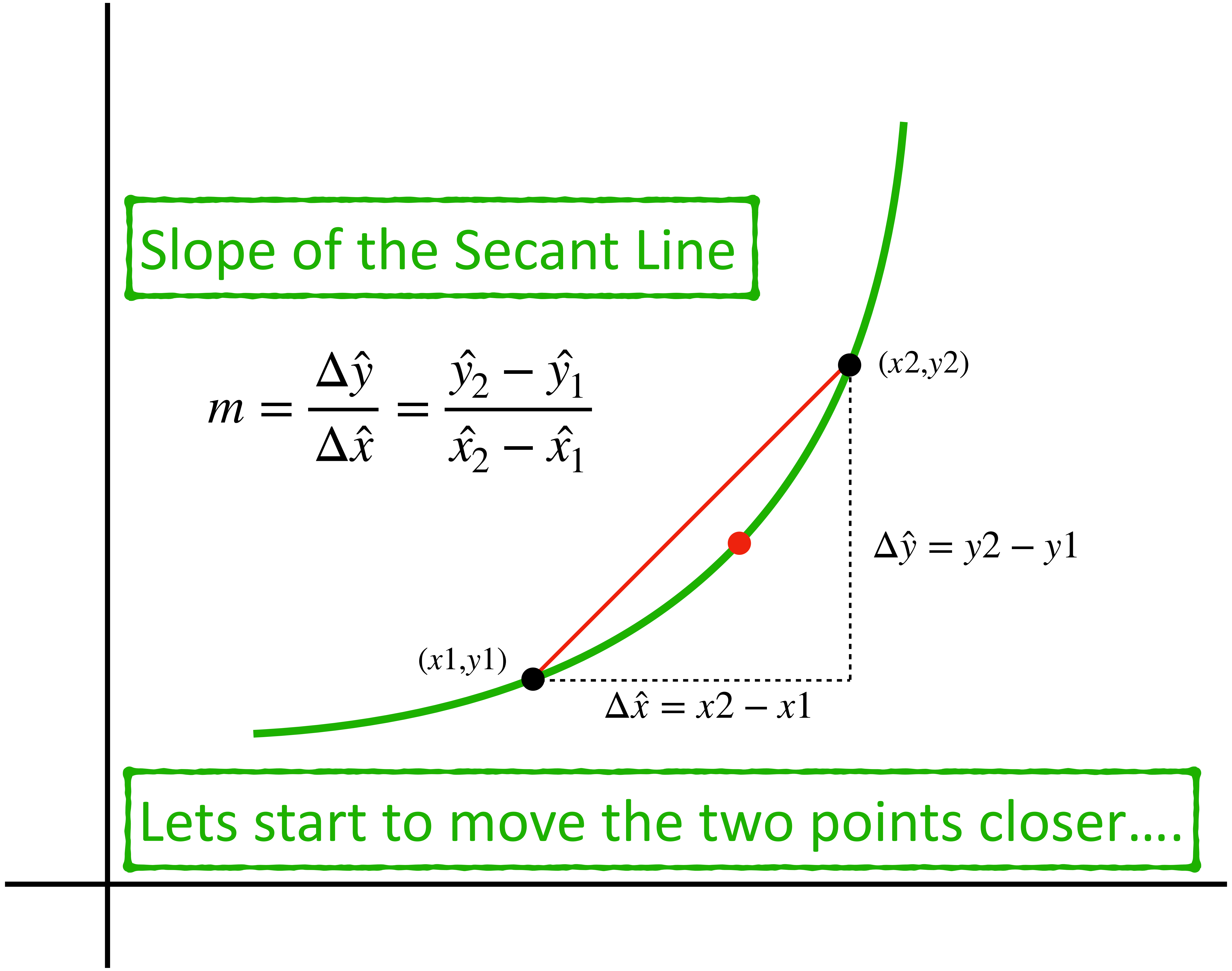

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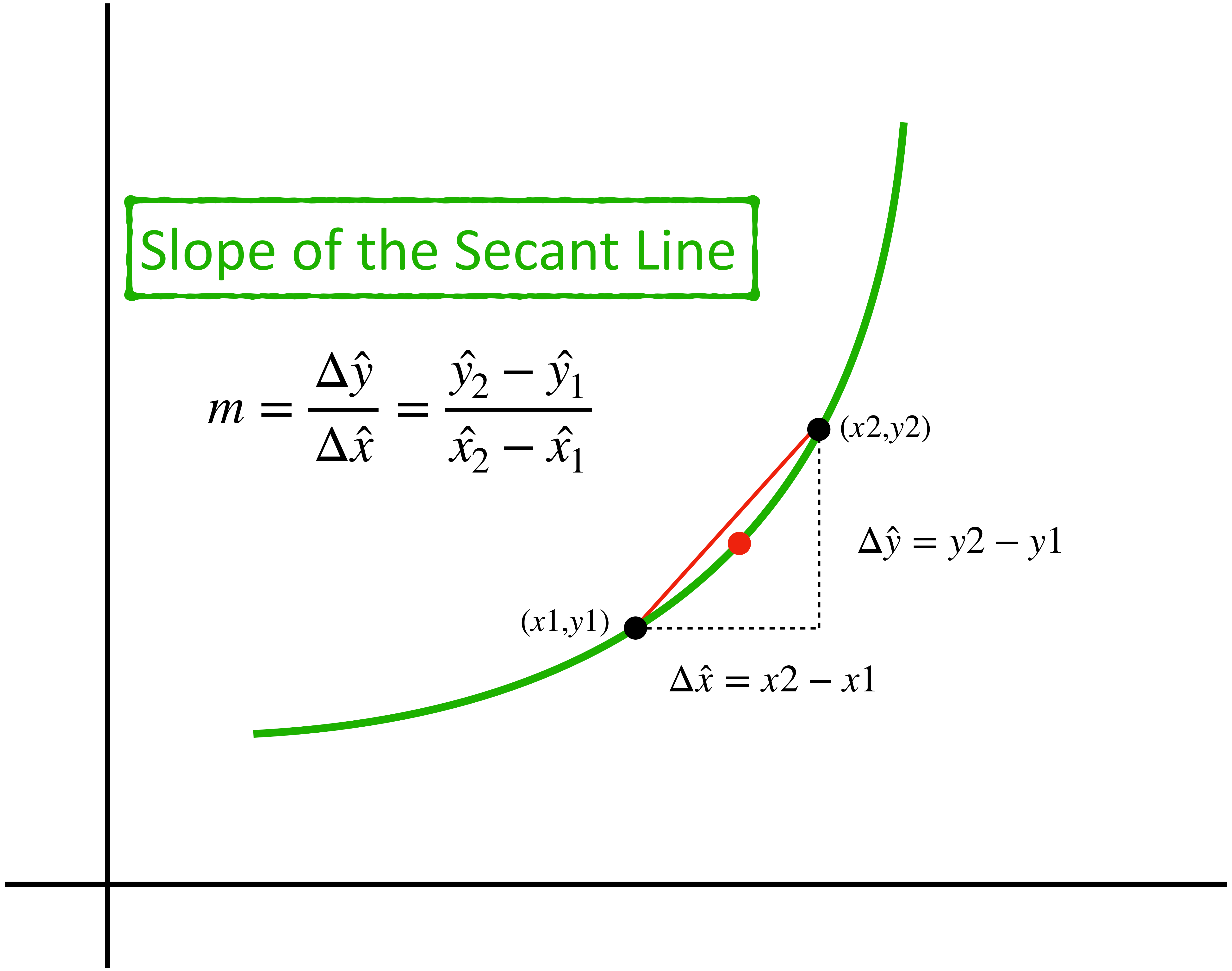

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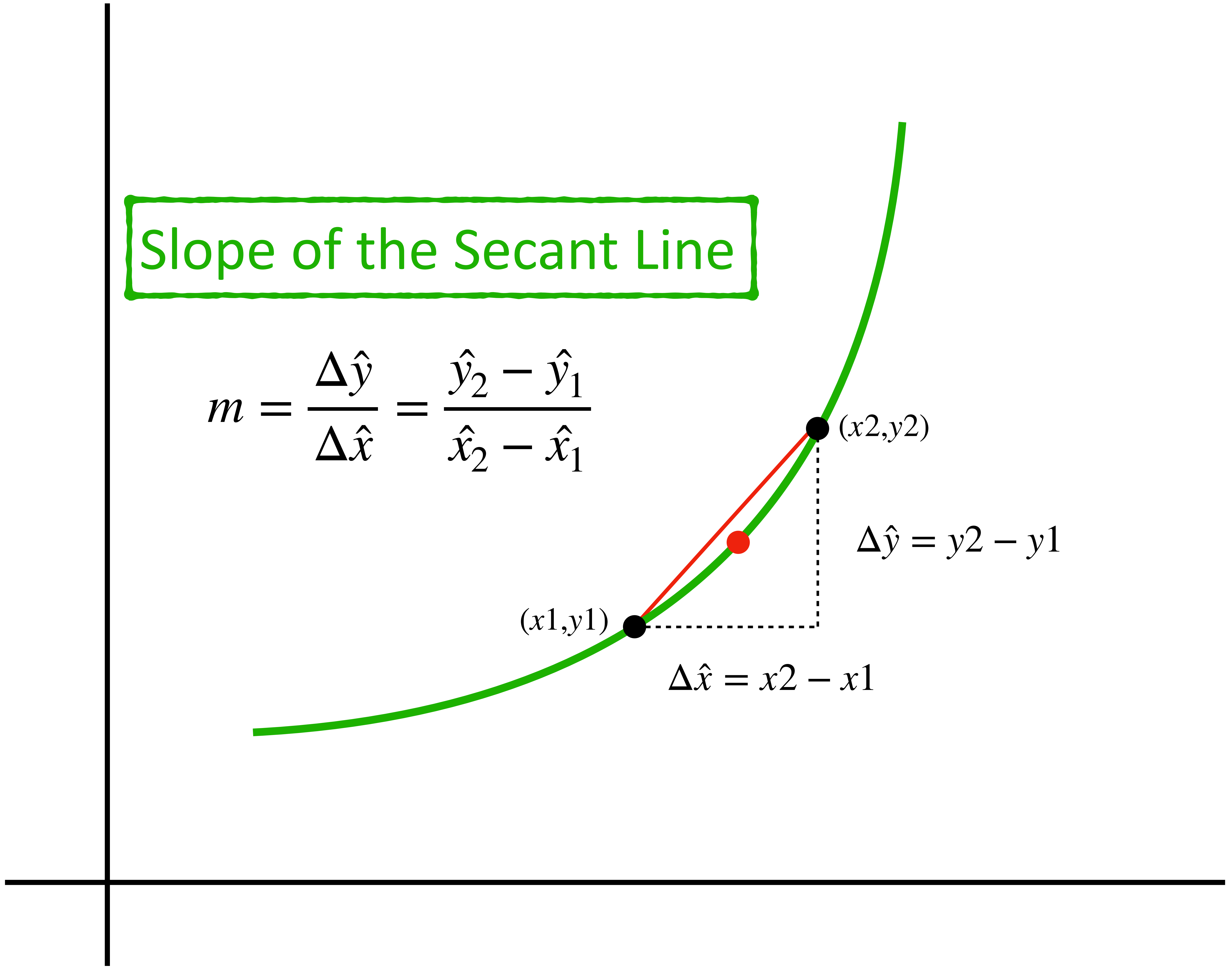
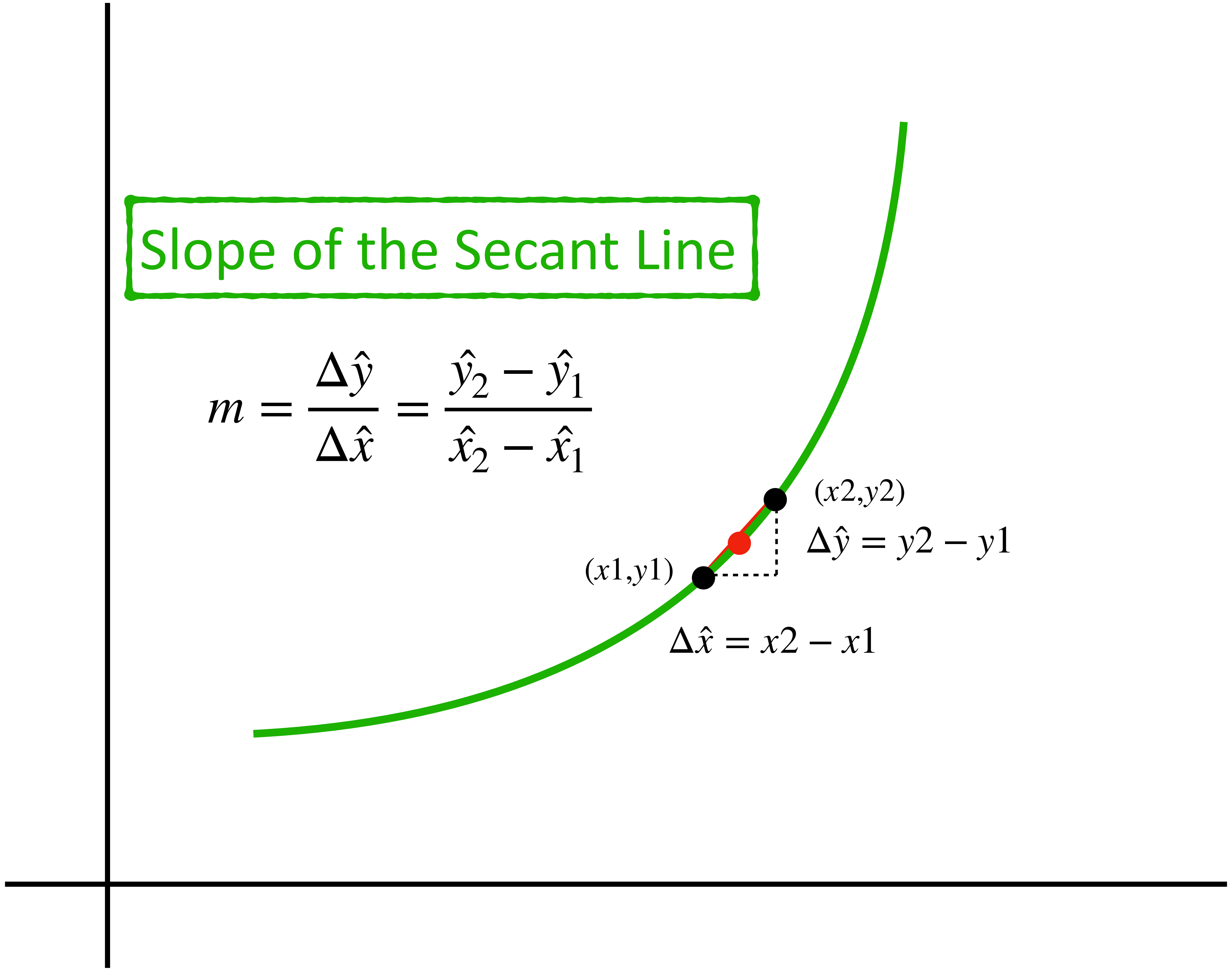


As the two points move closer, the value of Δx reduces

Slope of a Curve

Problem Statement: How do you calculate the slope of the tangent at a given point



As the two points move closer, the value of Δx tends to 0…

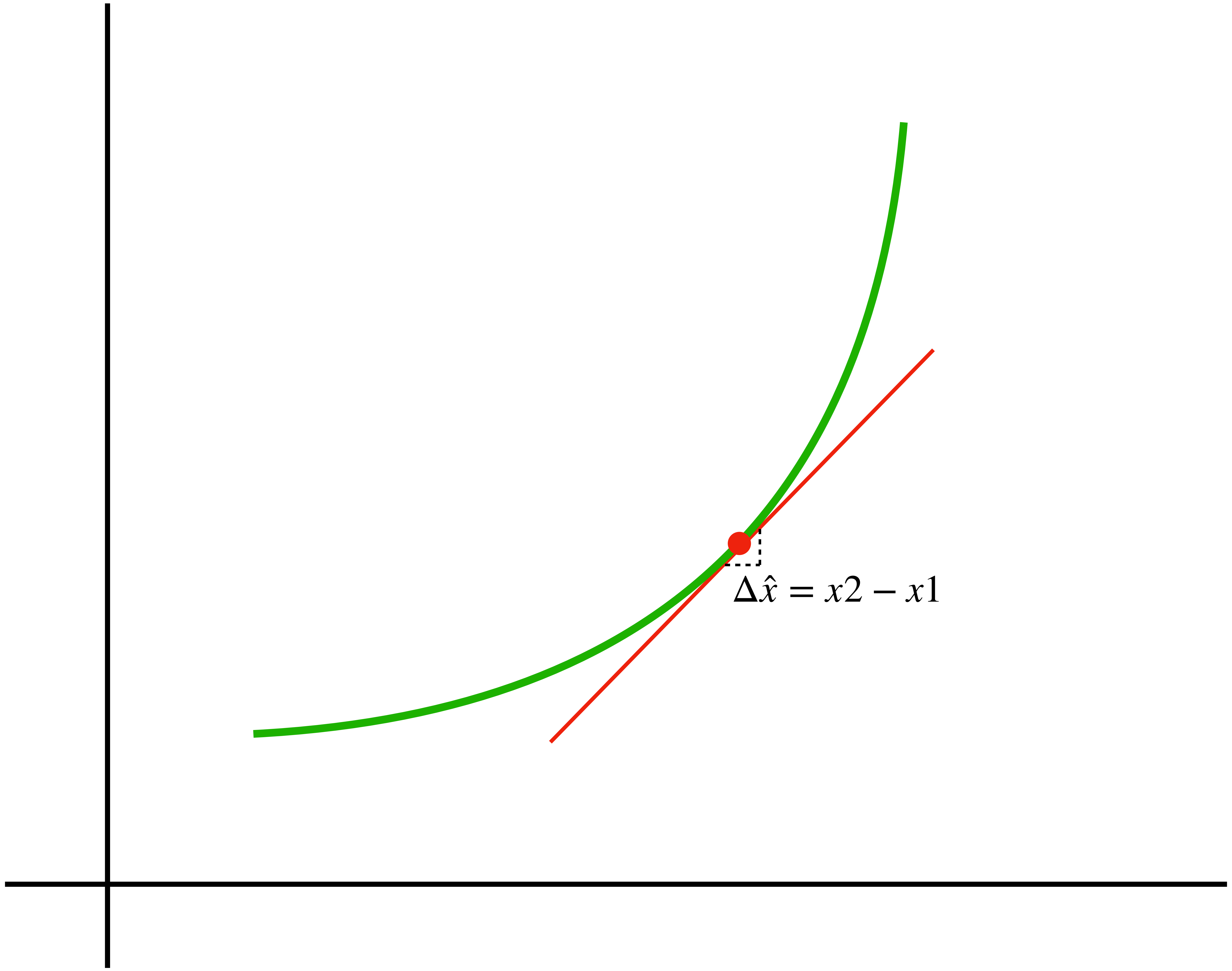
$$\Delta x \rightarrow 0$$

Slope of a Curve

Problem Statement: How do you calculate the slope of the tangent at a given point

Eventually the Secant line coincides with the Tangent line…

… and the slope of the Secant is the slope of the Tangent



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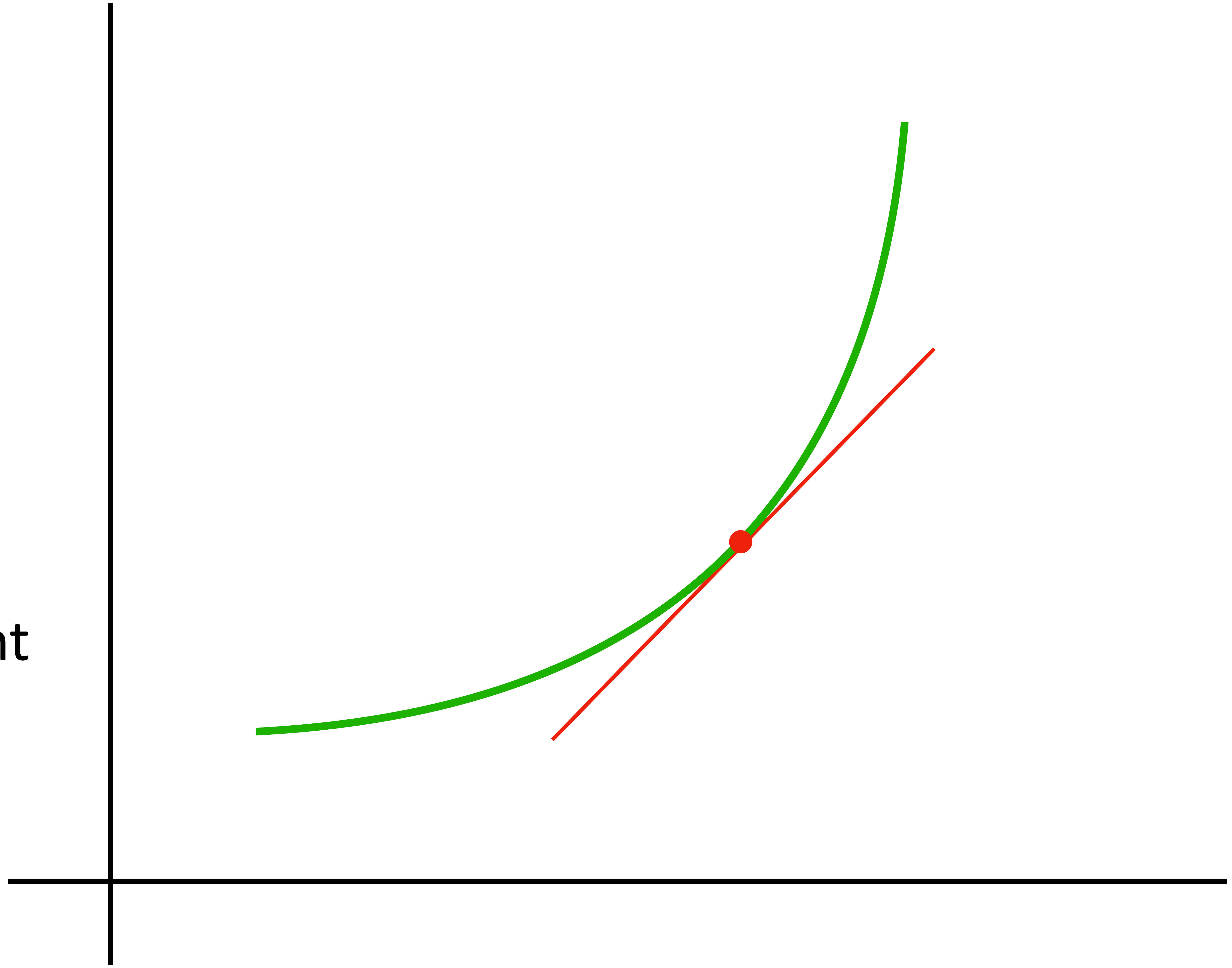
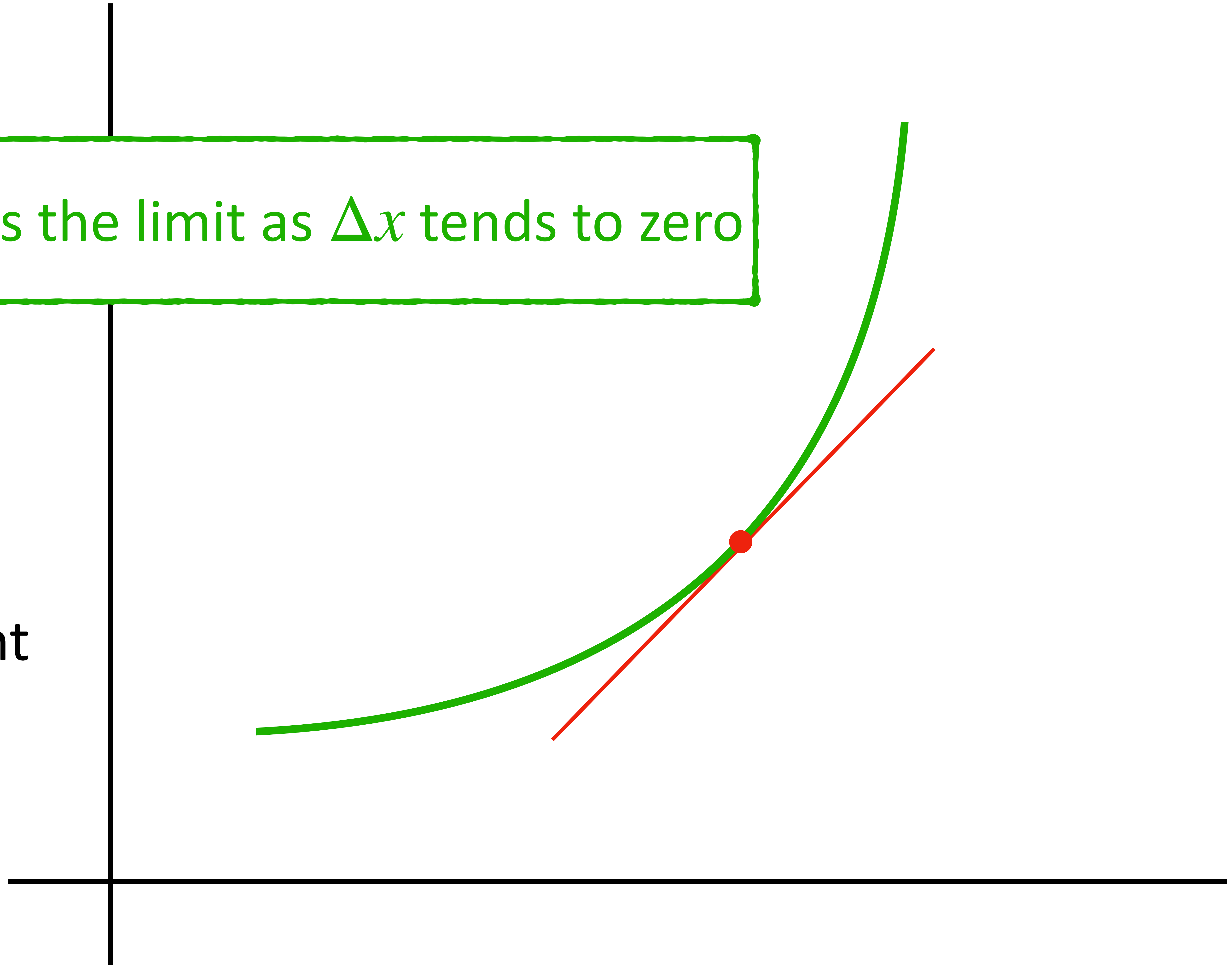


As $\Delta x \to 0$, the value of $\frac{\Delta y}{\Delta x}$ approaches the slope of the tangent to the curve at that point

Slope of a Curve

Problem Statement: How do you calculate the slope of the tangent at a given point

$\frac{\Delta y}{\Delta x}$ is the limit as Δx tends to zero



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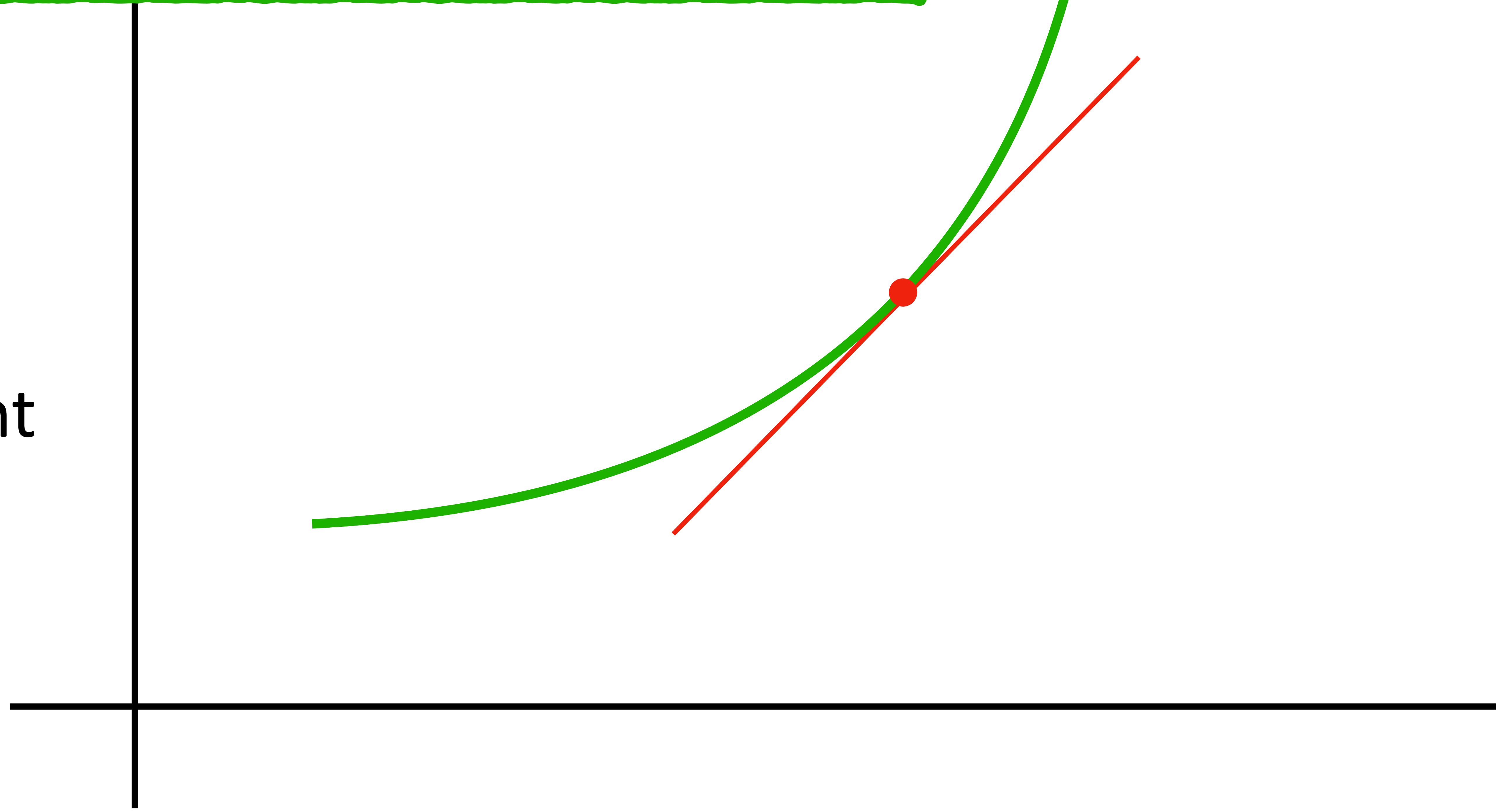
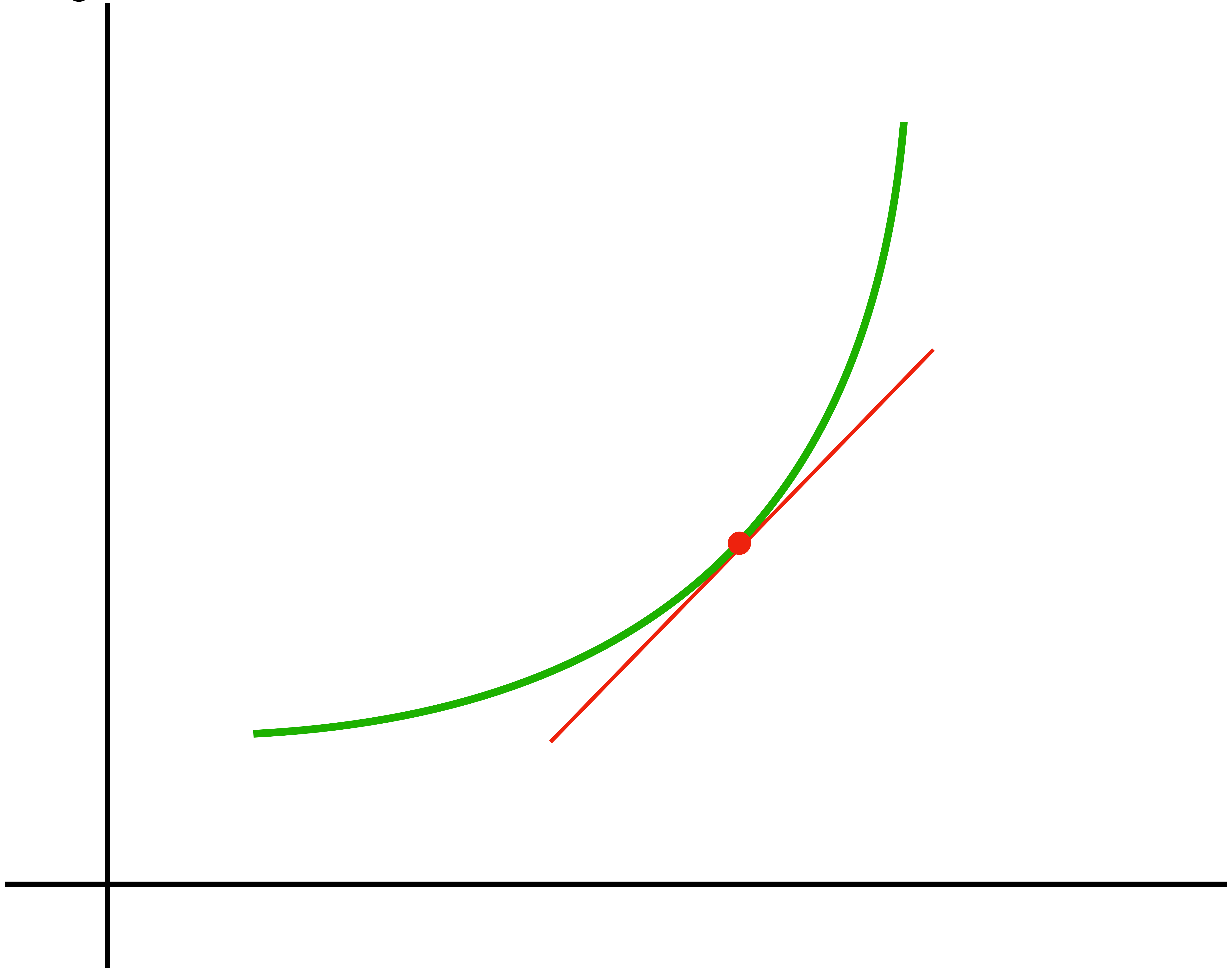


As $\Delta x \to 0$, the value of $\frac{\Delta y}{\Delta x}$ approaches the slope of the tangent to the curve at that point

This limit is called the **derivative** and represents the slope of the curve at that point

Slope of a Curve

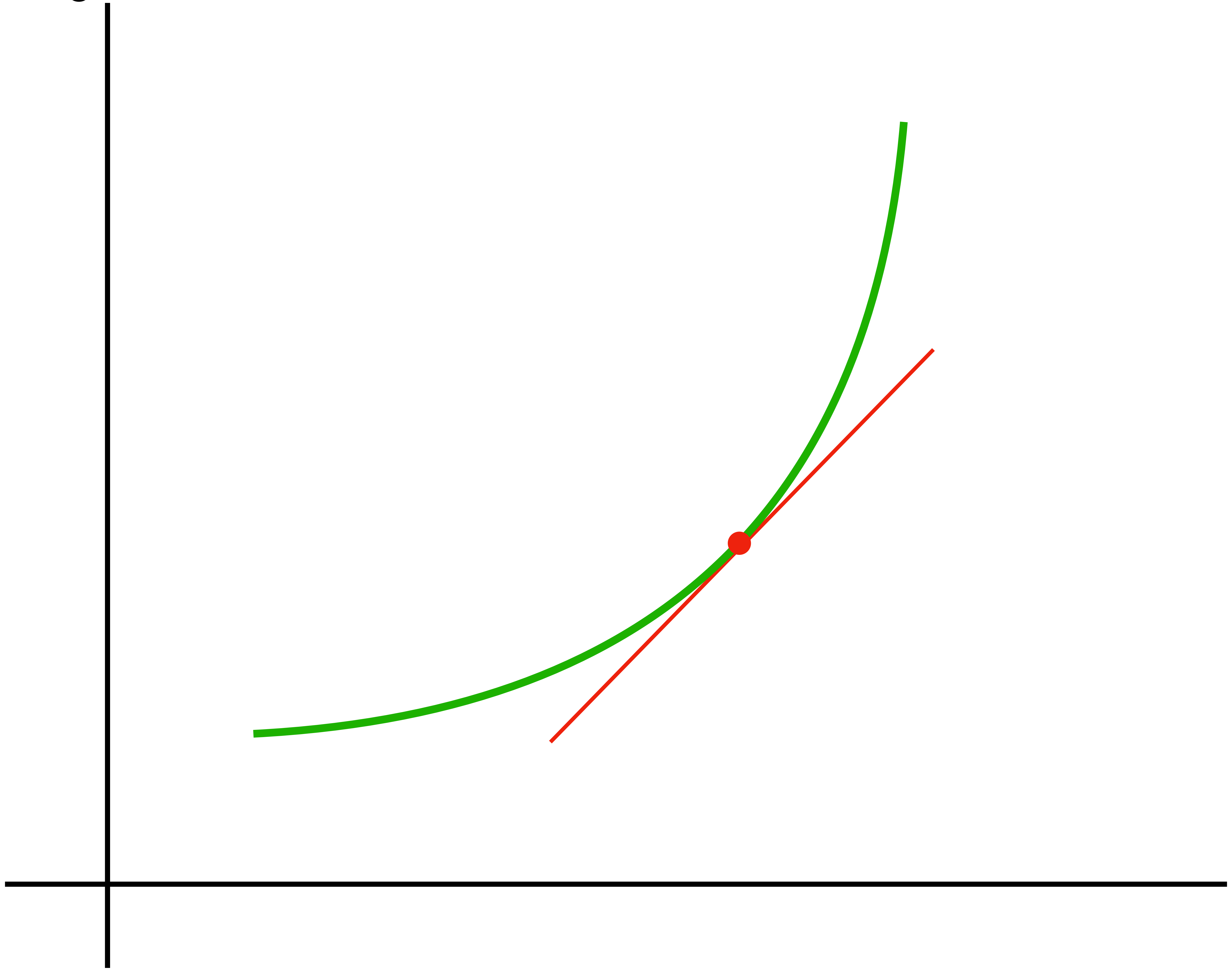
Derivative: Represents the slope of the tangent to the curve at a given point and is the limit as $\Delta x \to 0$



Slope of a Curve

Derivative: Represents the slope of the tangent to the curve at a given point and is the limit as $\Delta x \to 0$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$



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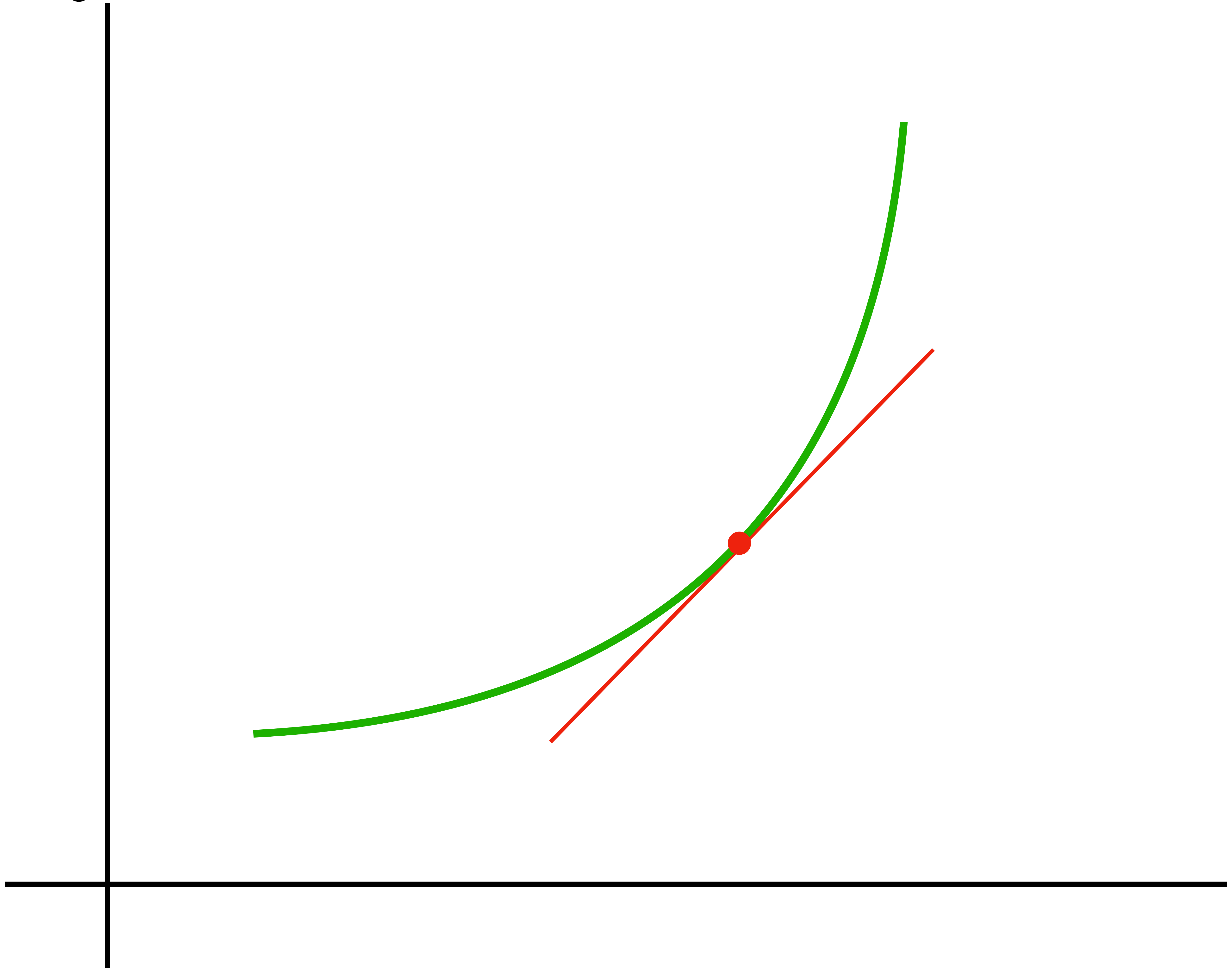
Lets take a real example…



Derivative: Derivative of $f(x) = x^2$

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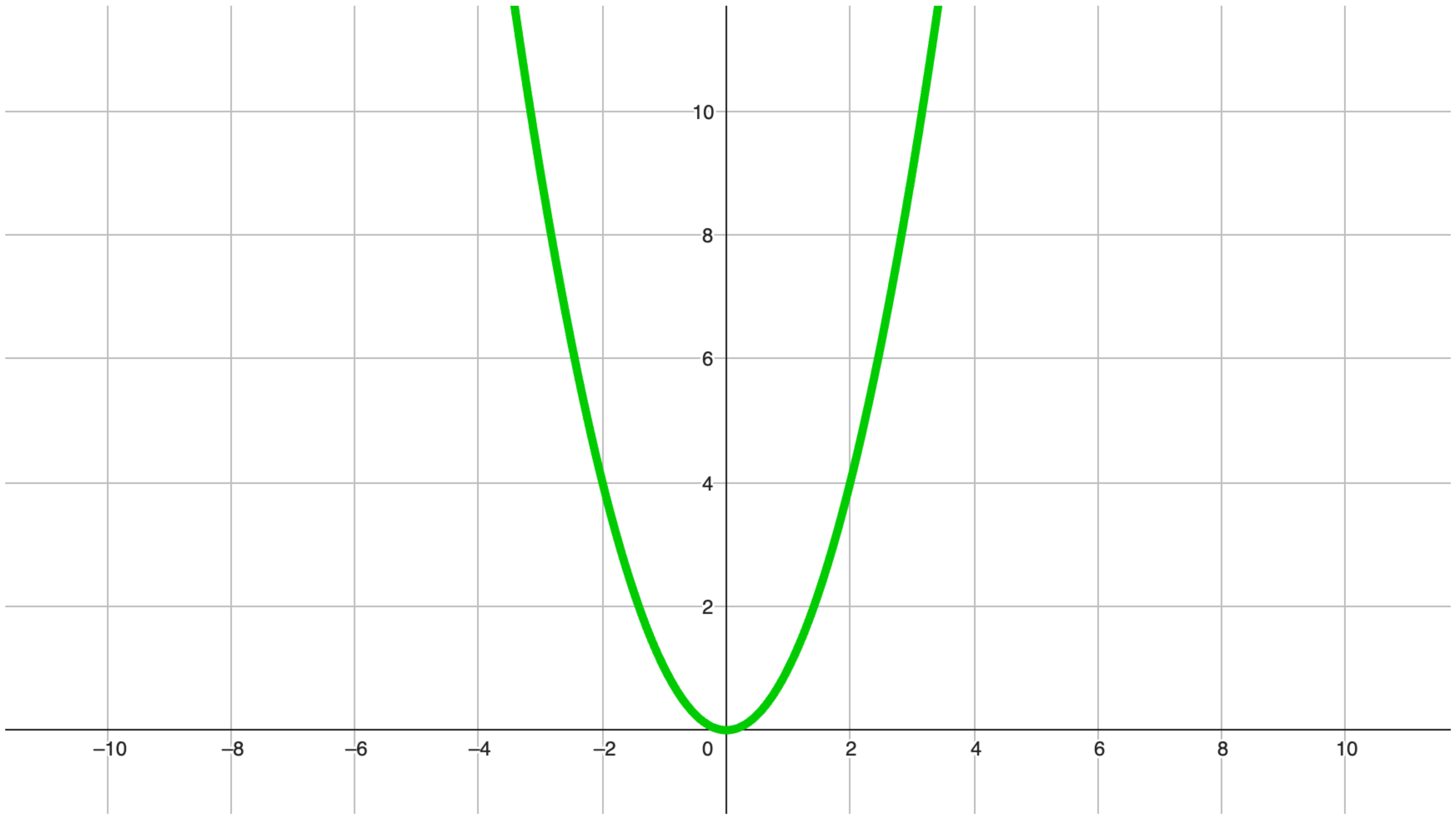

Slope of a Curve

Derivative: Derivative of $f(x) = x^2$

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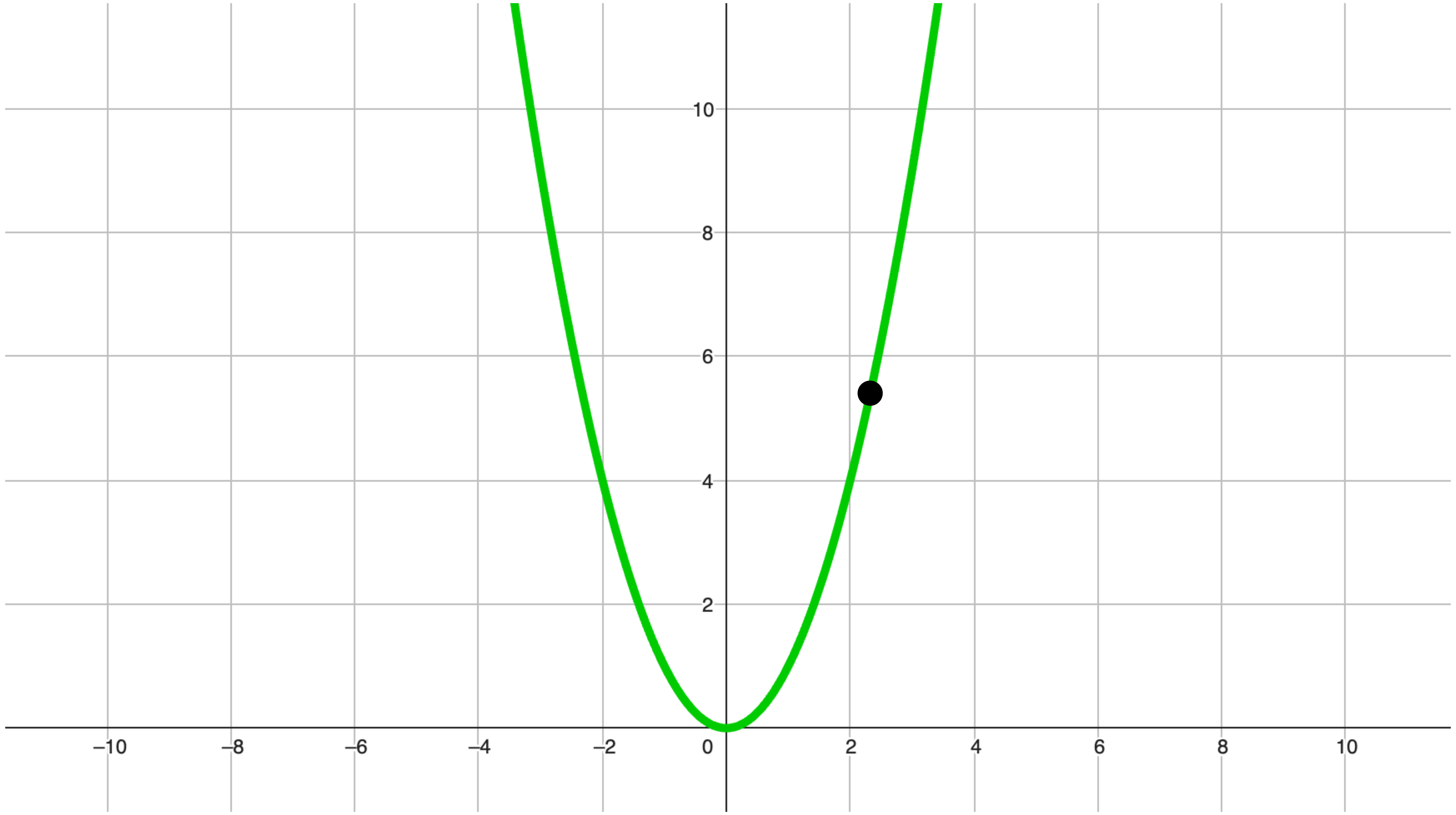
What is the slope of this curve at a given point?

$$f(x) = x^2$$



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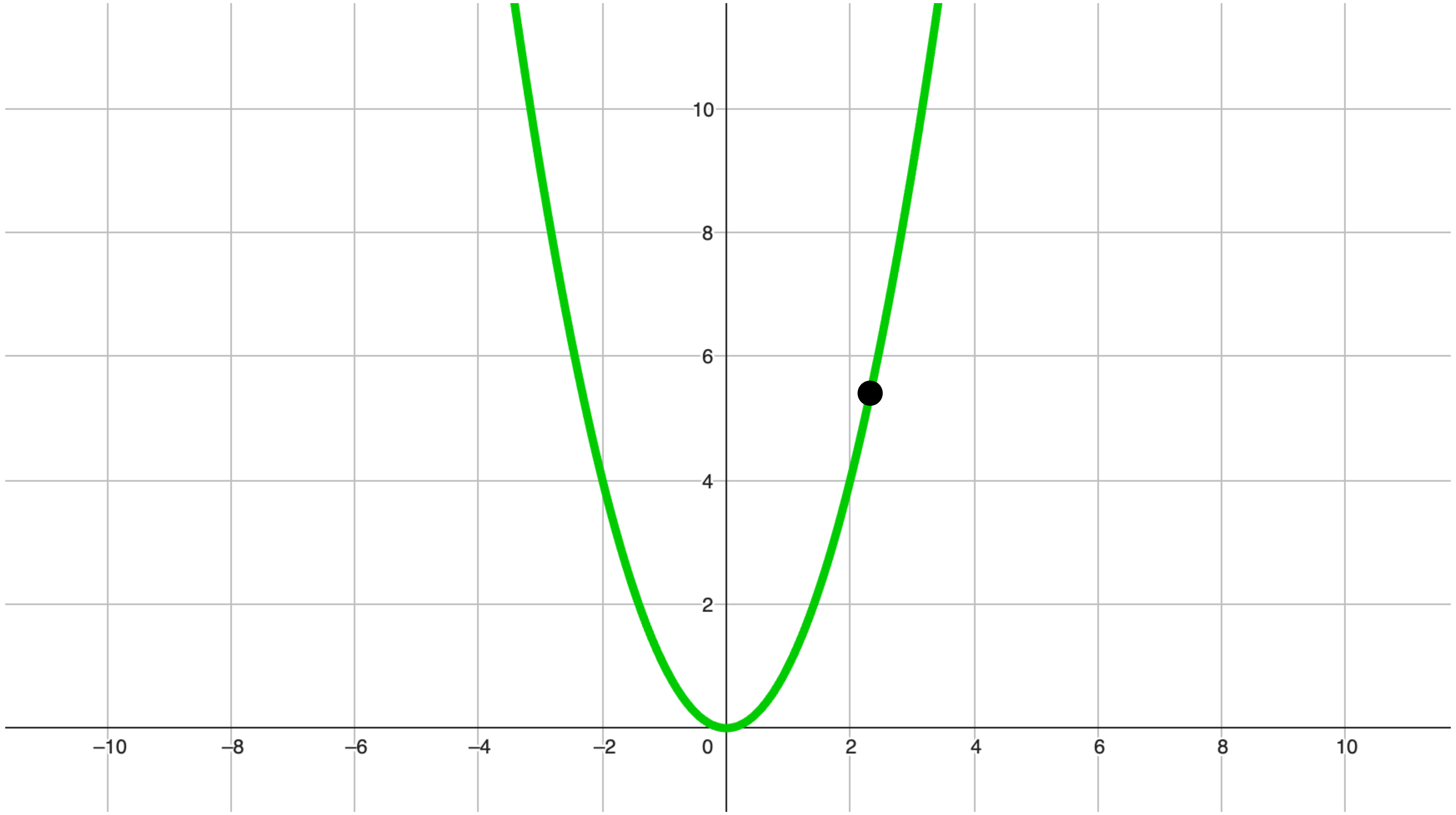

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$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

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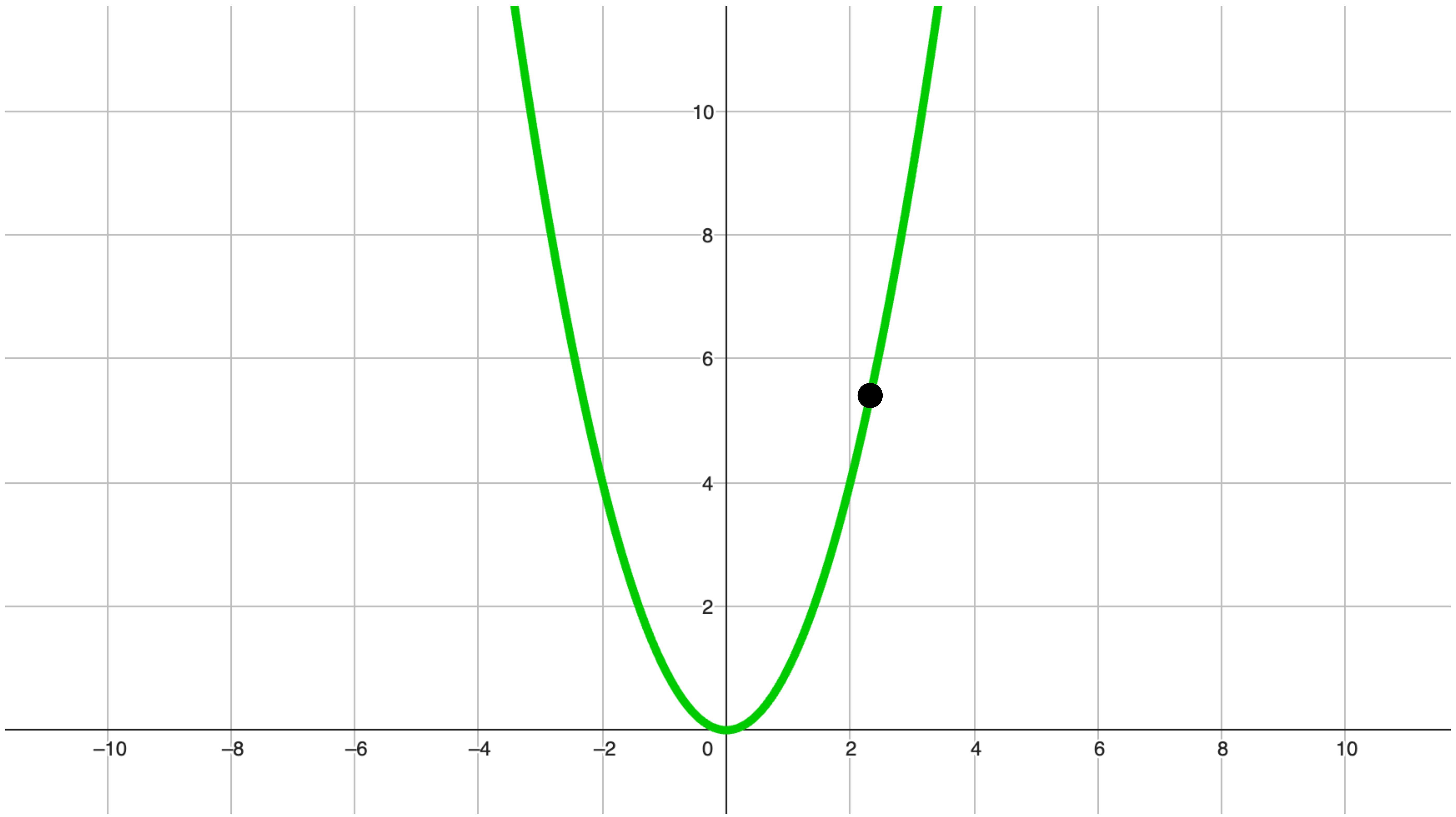

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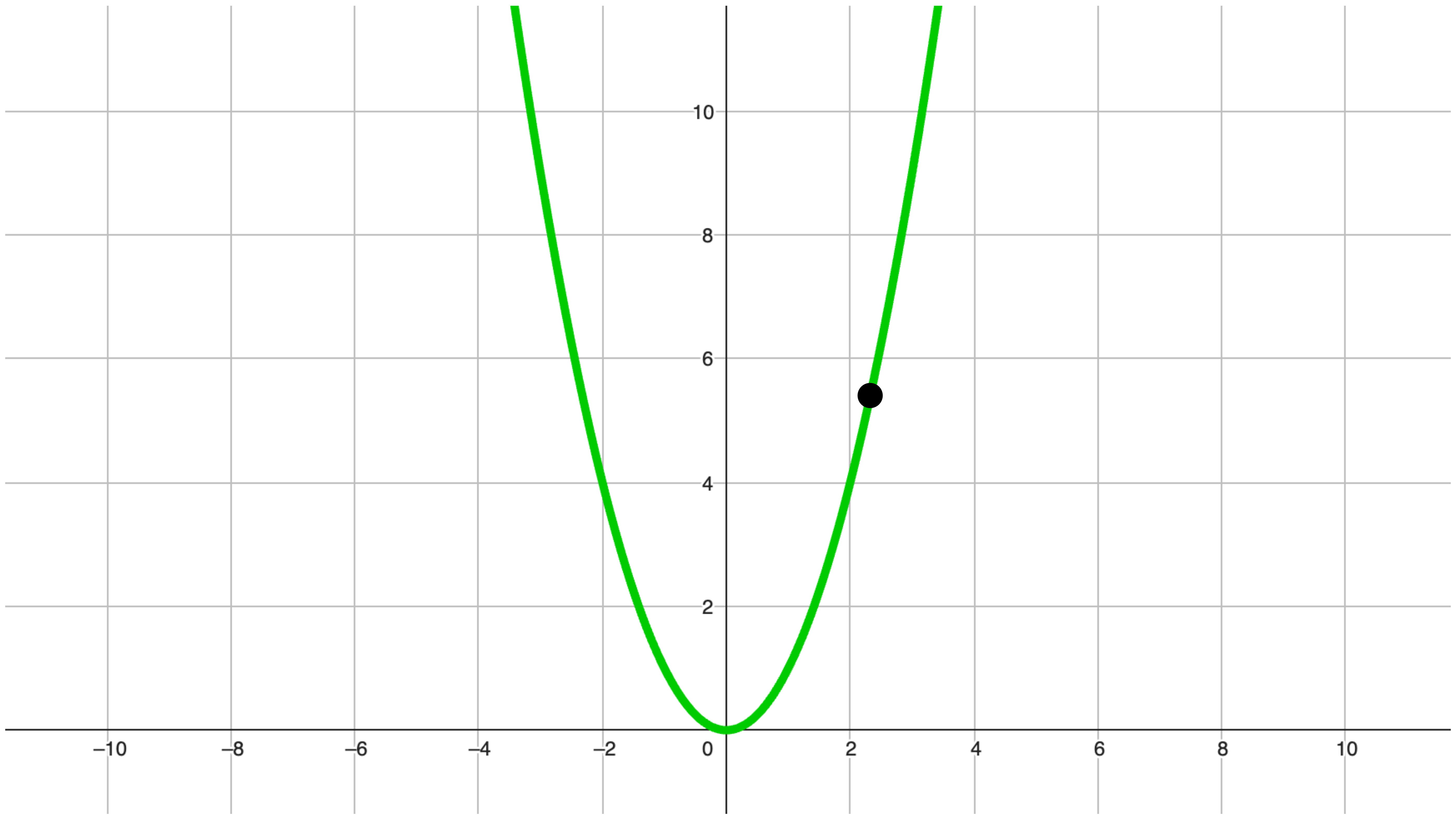

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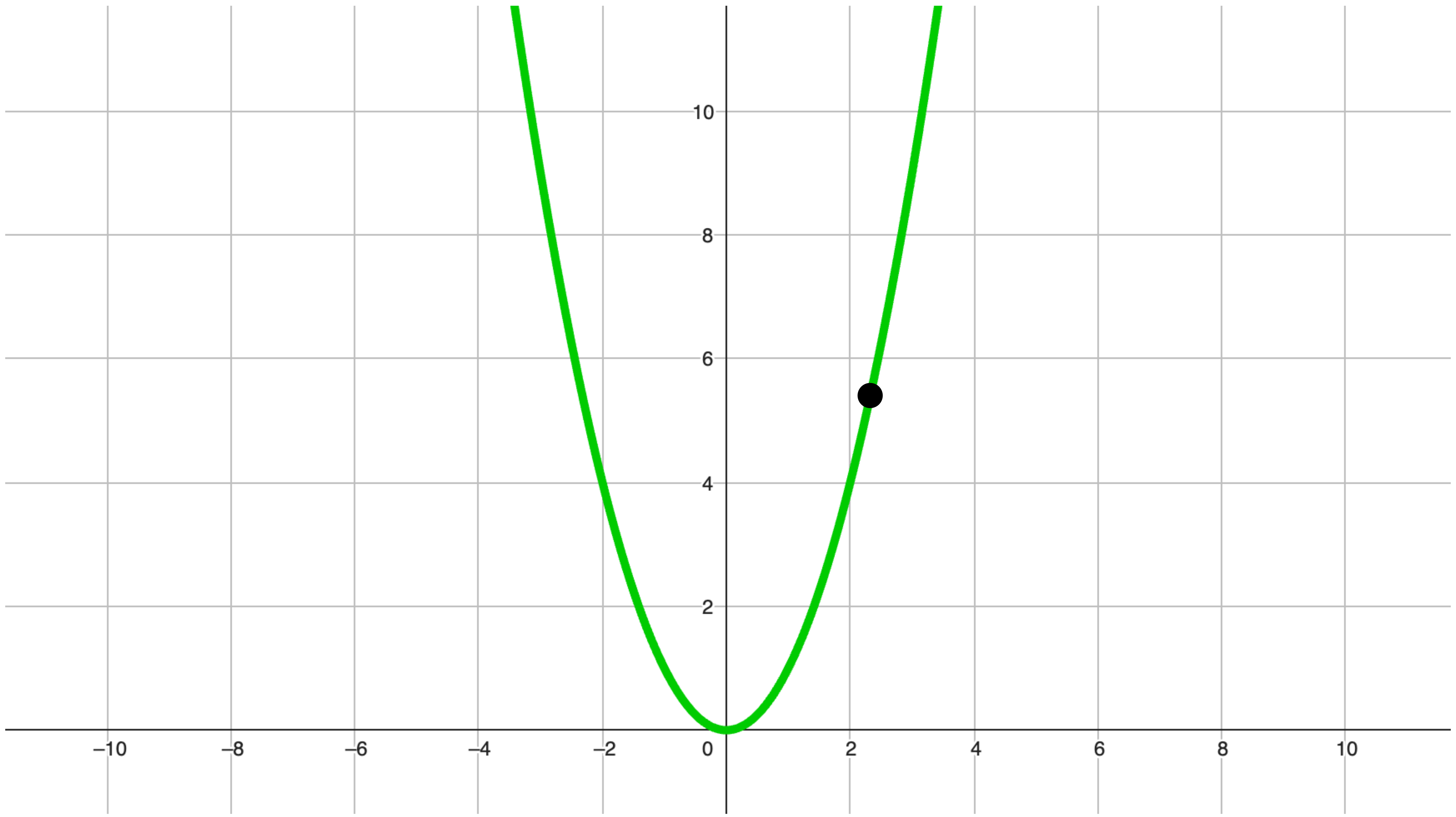

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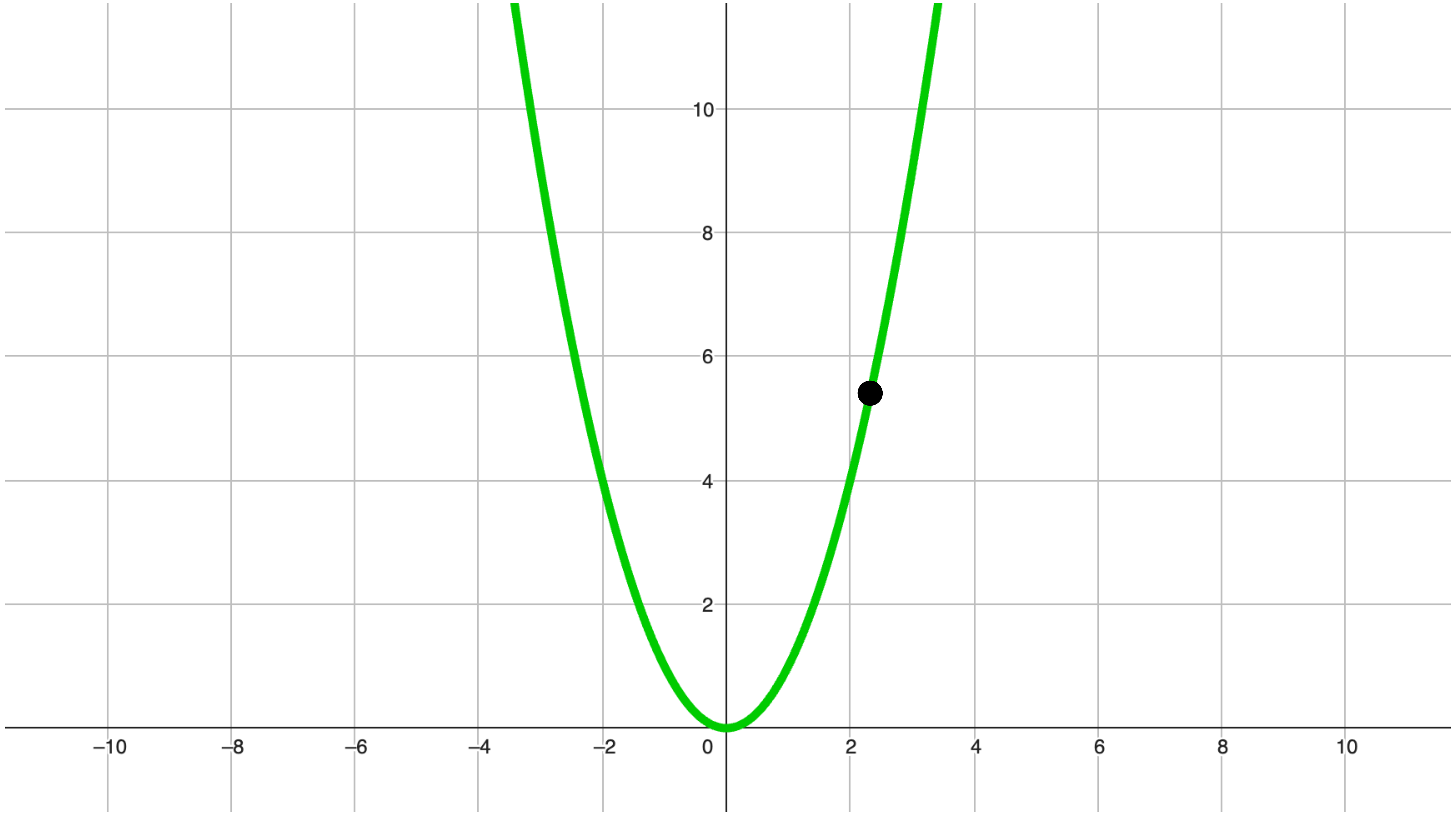

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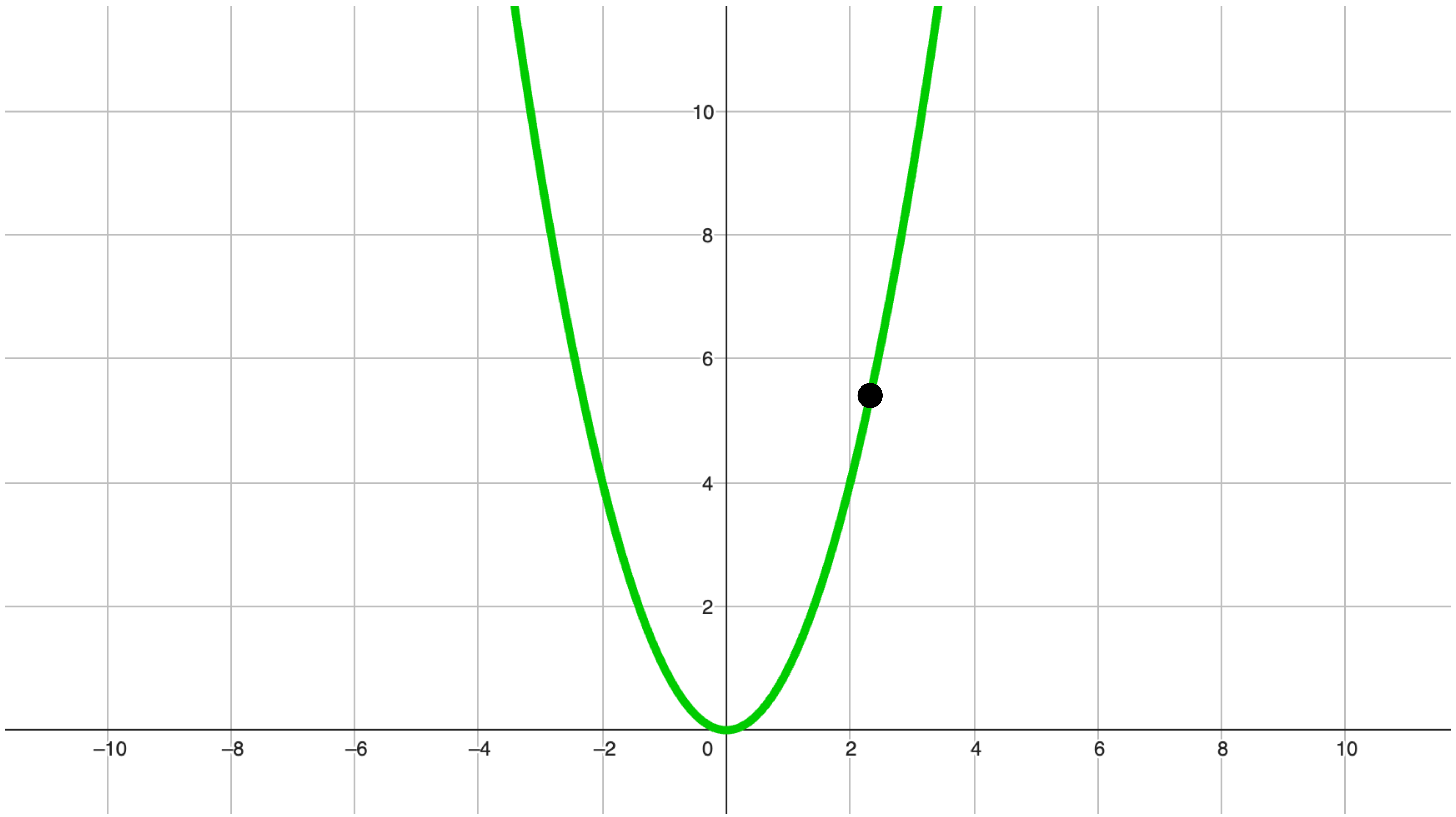

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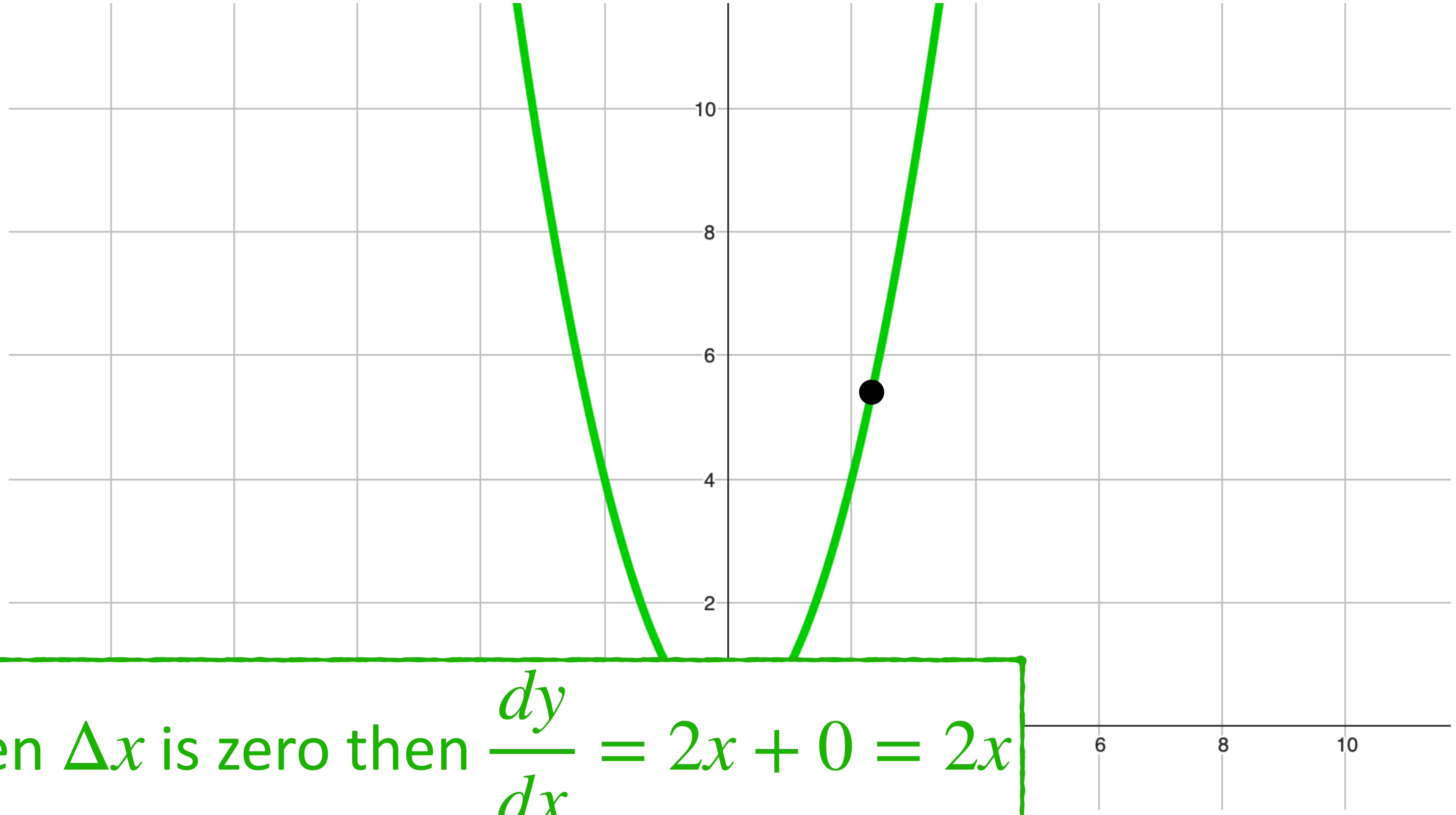

When Δx is zero then $\frac{dy}{dx} = 2x + 0 = 2x$

Derivative: Derivative of $f(x) = x^2$

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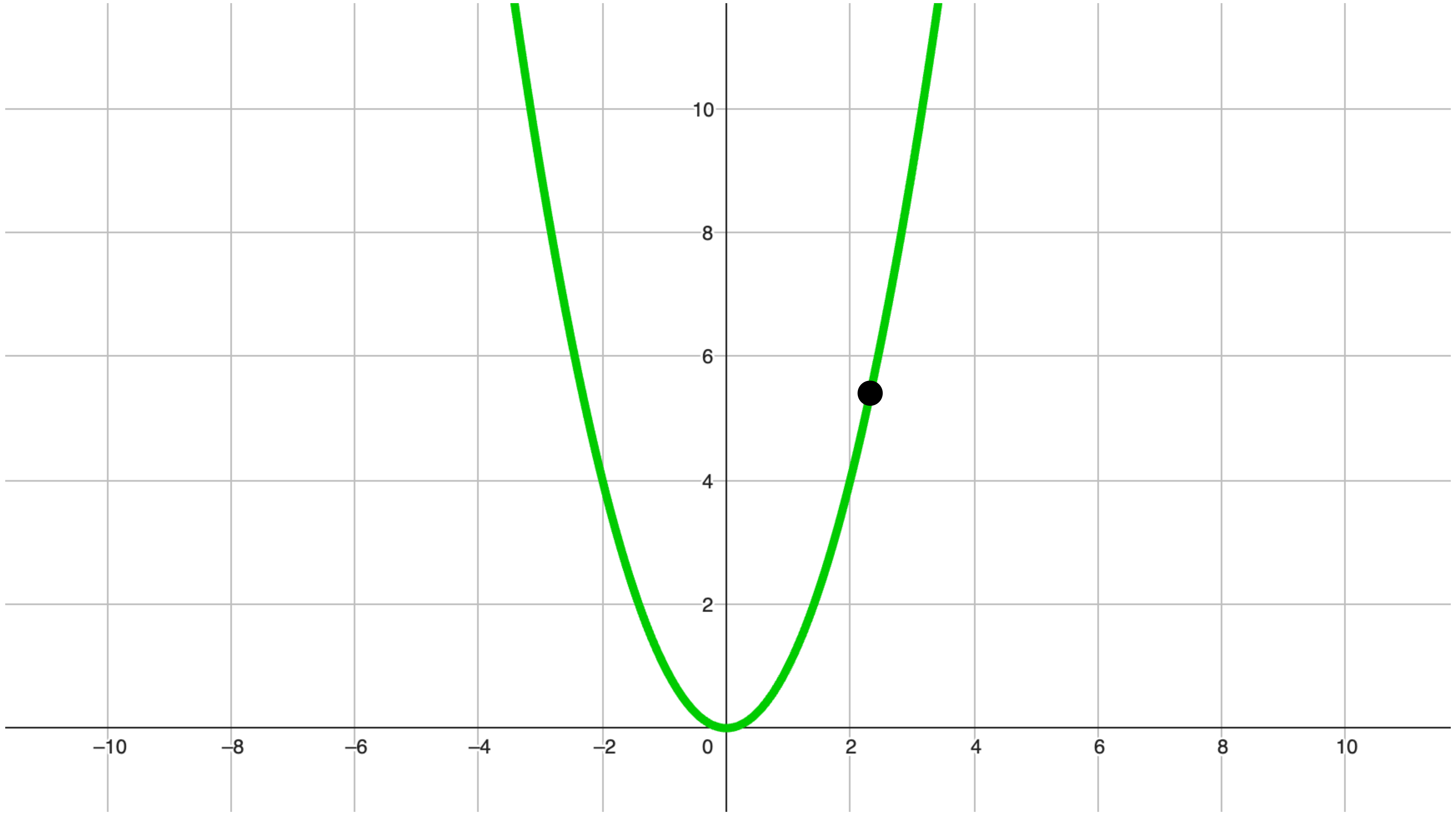


Derivative: Derivative of $f(x) = x^2$

The slope of the curve $y = x^2$ is the derivative of x^2

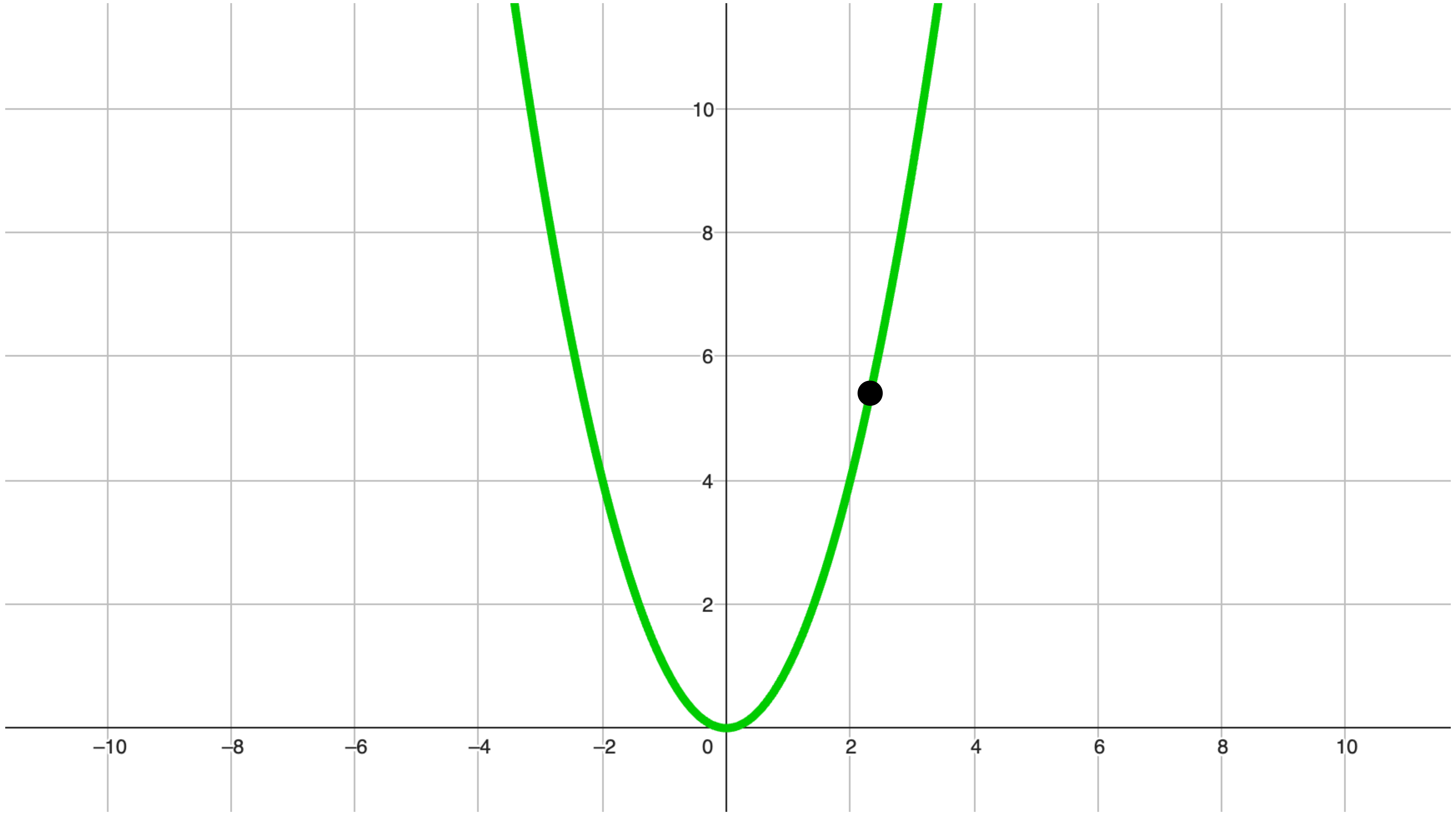
$$\frac{dy}{dx} = \frac{d}{dx}x^2 = 2x$$

When $x = 0$ the slope is 0

When $x = 1$ the slope is 2

When $x = 2$ the slope is 4

When $x = 12.9$ the slope is 25.8



Derivatives of some common functions

Derivative of a constant

$$\frac{d}{dx}C = 0$$

Derivative of a line

$$\frac{d}{dx}x = 1$$

Derivative of $y = x^2$

$$\frac{d}{dx}x^2 = 2x$$

Derivative of $y = \sqrt{x}$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2}x^{-\frac{1}{2}}$$

Derivative of $y = e^x$

$$\frac{d}{dx}e^x = e^x$$

Derivative of $y = e^{-x}$

$$\frac{d}{dx}e^{-x} = -\,e^{-x}$$

Derivative of $y = a^x$

$$\frac{d}{dx}a^x = log_e(a)\,a^x$$

Derivative of $y = log_e x$

$$\frac{d}{dx}log_e x = \frac{1}{x}$$

Derivative of $y = log_a x$

$$\frac{d}{dx}log_a x = \frac{1}{x\,log_e a}$$

Derivative of $y = sin(x)$

$$\frac{d}{dx}sin(x) = cos(x)$$

Derivative of $y = cos(x)$

$$\frac{d}{dx}cos(x) = -\,sin(x)$$

Derivative of $y = tan(x)$

$$\frac{d}{dx}tan(x) = sec^2(x)$$

Derivatives of some common functions

Derivative of $y = sin^{-1}x$

$$\frac{d}{dx}sin^{-1}x = \frac{1}{\sqrt{1 - x^2}}$$

Derivative of $y = cos^{-1}x$

$$\frac{d}{dx}cos^{-1}x = \frac{-1}{\sqrt{1 - x^2}}$$

Derivative of $y = tan^{-1}x$

$$\frac{d}{dx}tan^{-1}x = \frac{1}{1 + x^2}$$

Lets review some Derivative Rules

Multiplication by a Constant

$$\frac{d}{dx}Cf(x) = C\frac{d}{dx}f(x)$$

Example

$$\frac{d}{dx}4x^2 = 4\frac{d}{dx}x^2 = 4 \cdot 2x = 8x$$

Power Rule

$$\frac{d}{dx}x^n = n \cdot x^{(n-1)}$$

Example

$$\frac{d}{dx}x^4 = 4x^3$$

Sum Rule

$$\frac{d}{dx}\left(f(x)+g(x)\right)=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

Example

$$\frac{d}{dx}(x^3+x^5)=\frac{d}{dx}x^3+\frac{d}{dx}x^5=2x^2+5x^4$$

Difference Rule

$$\frac{d}{dx}\big(f(x) - g(x)\big) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Example

$$\frac{d}{dx}(x^3 - x^5) = \frac{d}{dx}x^3 - \frac{d}{dx}x^5 = 2x^2 - 5x^4$$

Product Rule

$$\frac{d}{dx} f(x)\, g(x) = f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$$

Example

$$\frac{d}{dx} 2x(x^2 + 3x) = 2x \frac{d}{dx}(x^2 + 3x) + (x^2 + 3x) \frac{d}{dx} 2x$$

$$\Rightarrow \frac{d}{dx} 2x(x^2 + 3x) = 2x(2x + 3) + 2(x^2 + 3x) = 4x^2 + 6x + 2x^2 + 6x = 6x^2 + 12x$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f(x)\frac{d}{dx}g(x) - g(x)\frac{d}{dx}f(x)}{g(x)^2}$$

Example

$$\frac{d}{dx}\left(\frac{x^2}{sin(x)}\right) = \frac{x^2\frac{d}{dx}sin(x) - sin(x)\frac{d}{dx}x^2}{sin^2(x)} = \frac{x^2cos(x) - 2x\,sin(x)}{sin^2(x)}$$

Reciprocal Rule

$$\frac{d}{dx}\left(\frac{1}{f(x)}\right) = \frac{-1}{f(x)^2}\frac{d}{dx}f(x)$$

Example

$$\frac{d}{dx}\left(\frac{1}{sin(x)}\right) = \frac{-1}{sin^2(x)}\frac{d}{dx}sin(x) = \frac{-cos(x)}{sin^2(x)}$$

Chain Rule

$$\frac{d}{dx} f(g(x)) = \frac{d}{dg} f(g) \frac{d}{dx} g(x)$$

Example

$$\frac{d}{dx} sin(x^2) = \frac{d}{dz} sin(z) \frac{d}{dx} x^2 = cos(x^2)\, 2x = 2x\, cos(x^2)$$

Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Example

$$\frac{d}{dx} sin(x^2) = \frac{d}{dz} sin(z) \frac{d}{dx} x^2 = cos(x^2)\, 2x = 2x\, cos(x^2)$$

Partial Derivative

Given a function with multiple variables…

$$f(x, y, z)$$

…the partial derivative is the derivative w.r.t one of the variables, keeping the others constant

Example:

$$f(x, y, z) = 3x^2 + 4y^3 + 7z^4$$

Partial Derivative w.r.t x is…

$$\frac{\partial}{\partial x} f(x, y, z) = \frac{\partial}{\partial x}(3x^2 + 4y^3 + 7z^4) = 6x$$

$4y^3$ is a constant so $\frac{\partial}{\partial x} 4y^3 = 0$

$7z^4$ is a constant so $\frac{\partial}{\partial x} 7z^4 = 0$

Partial Derivative

Given a function with multiple variables…

$$f(x, y, z)$$

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Example:

$$f(x, y, z) = 3x^2 + 4y^3 + 7z^4$$

Partial Derivative w.r.t y is…

$$\frac{\partial}{\partial y} f(x, y, z) = \frac{\partial}{\partial y}(3x^2 + 4y^3 + 7z^4) = 12y^2$$

$3x^2$ is a constant so $\frac{\partial}{\partial y}3x^2 = 0$

$7z^4$ is a constant so $\frac{\partial}{\partial y}7z^4 = 0$

Partial Derivative

Given a function with multiple variables…

$$f(x, y, z)$$

…the partial derivative is the derivative w.r.t one of the variables, keeping the others constant

Example:

$$f(x, y, z) = 3x^2 + 4y^3 + 7z^4$$

Partial Derivative w.r.t z is…

$$\frac{\partial}{\partial z} f(x, y, z) = \frac{\partial}{\partial z}(3x^2 + 4y^3 + 7z^4) = 28z^3$$

$3x^2$ is a constant so $\frac{\partial}{\partial z} 3x^2 = 0$

$4y^3$ is a constant so $\frac{\partial}{\partial z} 4y^3 = 0$

Related Tutorials & Textbooks

Logistic Regression

An introduction to Logistic Regression. A Logistic Regression model use used to predict a binary value (the dependent variable) for one or more independent variables using a threshold to classify a probability.

Multiple Regression

Multiple regression extends the two dimensional linear model introduced in Simple Linear Regression to k + 1 dimensions with one dependent variable, k independent variables and k+1 parameters.

Cost Function & Gradient Descent for Logistic Regression

An introduction to the Cost function for Logistic Regression long with its partial derivative (the gradient vector). The model parameters (B & W) are then optimized using Maximum Likelihood Estimation and Gradient Descent.

For a complete list of tutorials see:
https://arrsingh.com/ai-tutorials