

Probability & Statistics

Fundamentals

Rahul Singh
rsingh@arrsingh.com

Experiments & Events

Experiment (aka Trial)

Any procedure that can be repeated infinitely and has a well defined set of outcomes

Random Experiment (aka Trial)

If the experiment can have more than one possible outcome, then its a random experiment

Event

An event is the subset of the outcomes of an experiment

Example

Experiment: Flipping a fair coin (this is a random event)

Outcomes: This experiment has two outcomes: Heads or Tails

Event: The coin lands with heads on top

Random Variable

A numeric quantity associated with random events.

Random variable maps the set of possible outcomes (the domain) to a numeric quantity (the range)

Discrete Random Variable

A random variable with a finite, countable, distinct set of outcomes

Mathematical Representation

Flipping a fair coin

$$X(\alpha) = \begin{cases} 1, & \text{if } \alpha = \text{Heads} \\ 0, & \text{if } \alpha = \text{Tails} \end{cases}$$

Two Possible Outcomes: Heads or Tails.
Heads is mapped to 1 and Tails is mapped to 0

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X is a discrete random variable

Two Possible Outcomes: Heads or Tails.
Heads is mapped to 1 and Tails is mapped to 0

Probability of an Event Occurring

The number of desired outcomes divided by the total number of outcomes

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$$P(X = 1) = \frac{1}{2} = 0.5$$

Probability that the coin lands on Heads ($X = 1$)

$$P(X = 1) = \frac{\text{Number of Desired Outcomes}}{\text{Total Number of Outcomes}} = \frac{1}{2} = 0.5$$

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We can plot these probabilities on a graph

Probability of an Event Occurring

The number of desired outcomes divided by the total number of outcomes

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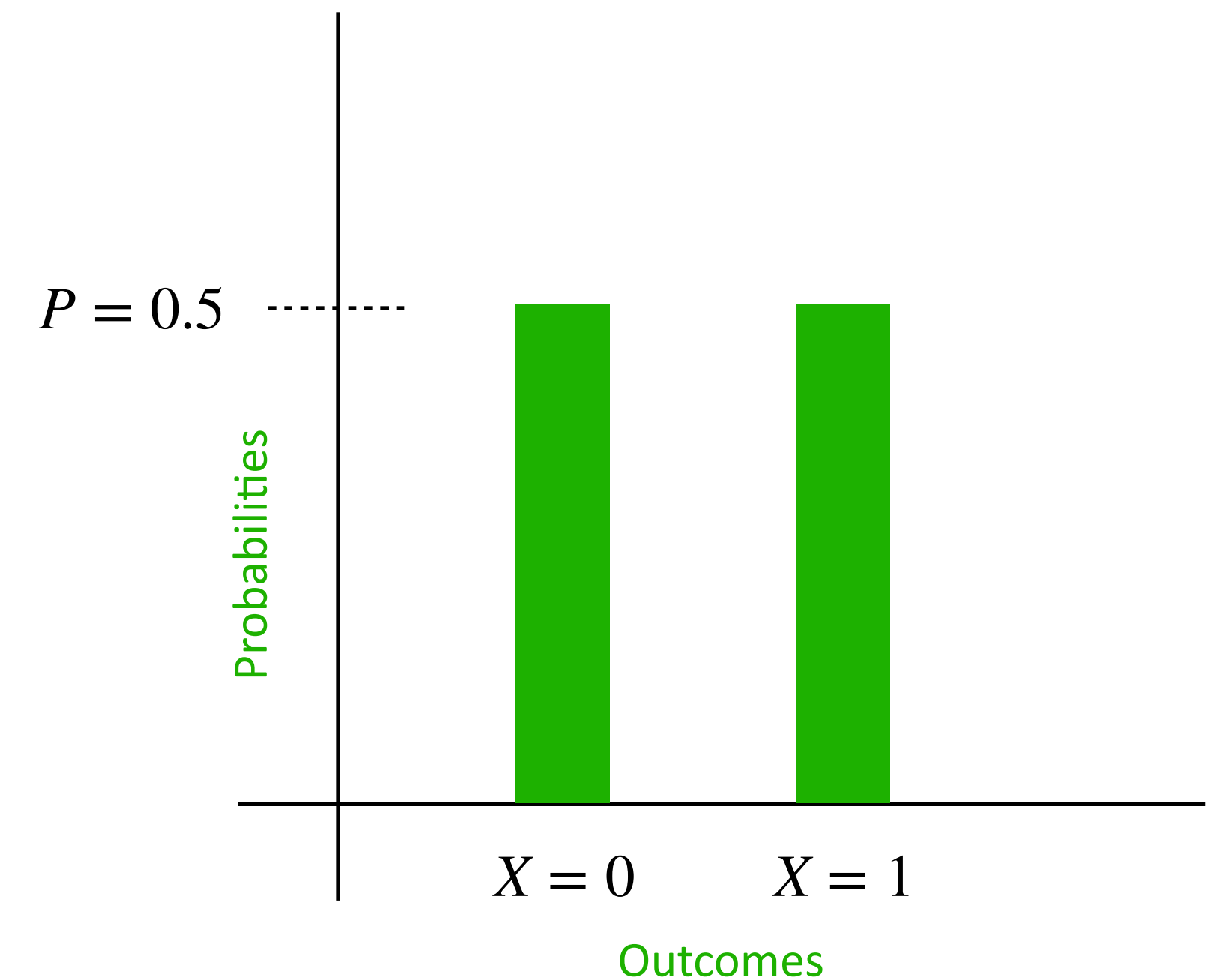
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Probability Mass Function

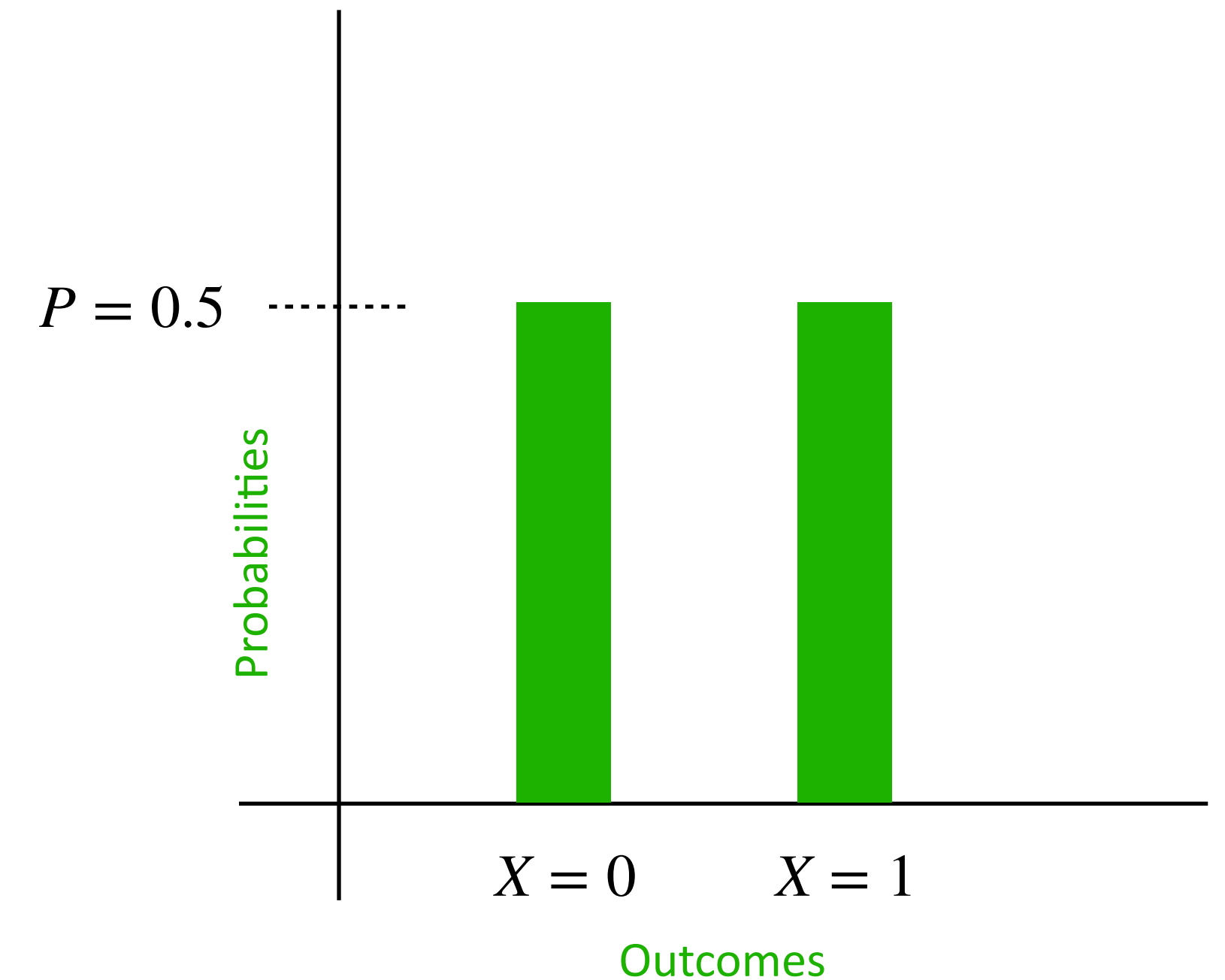
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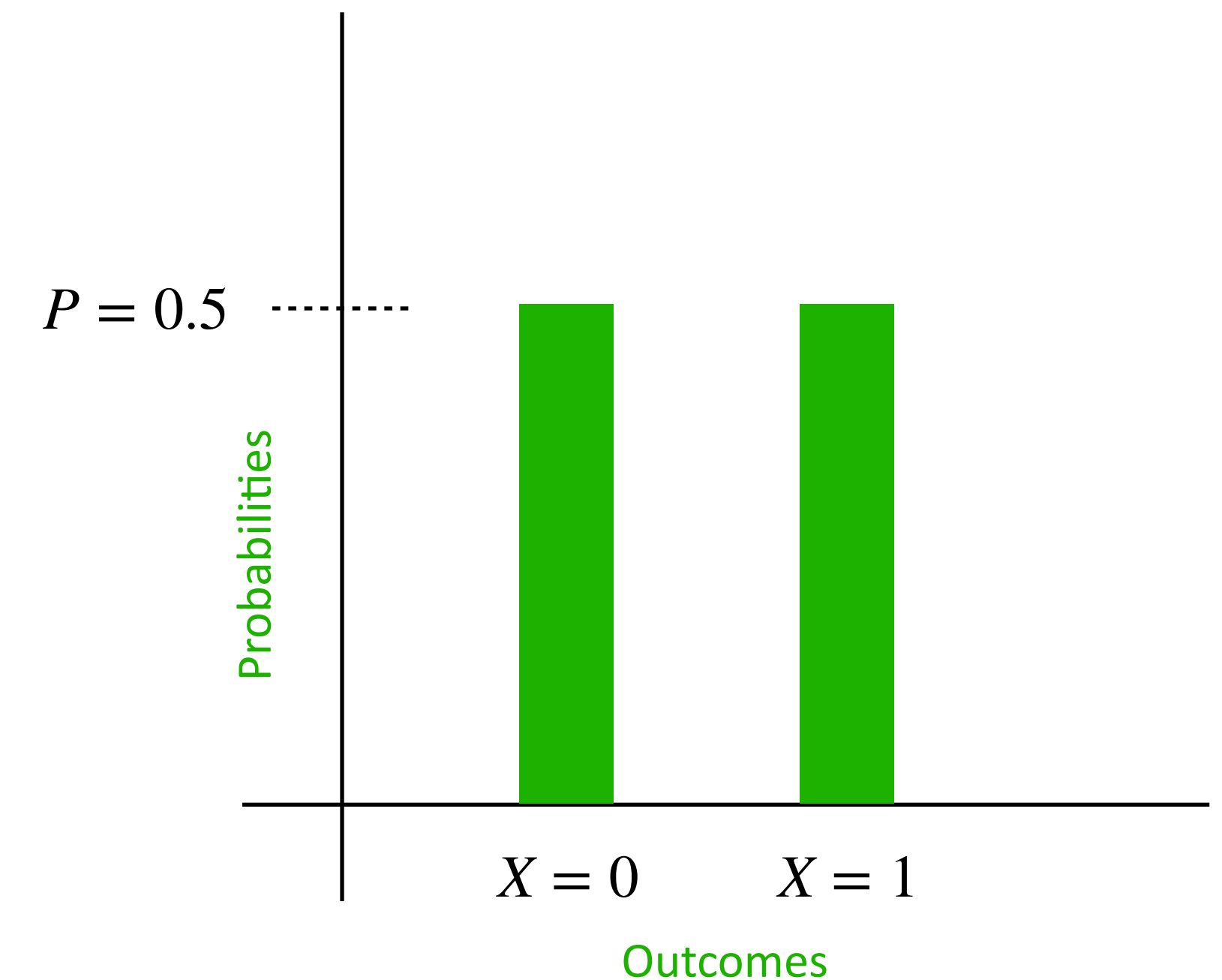
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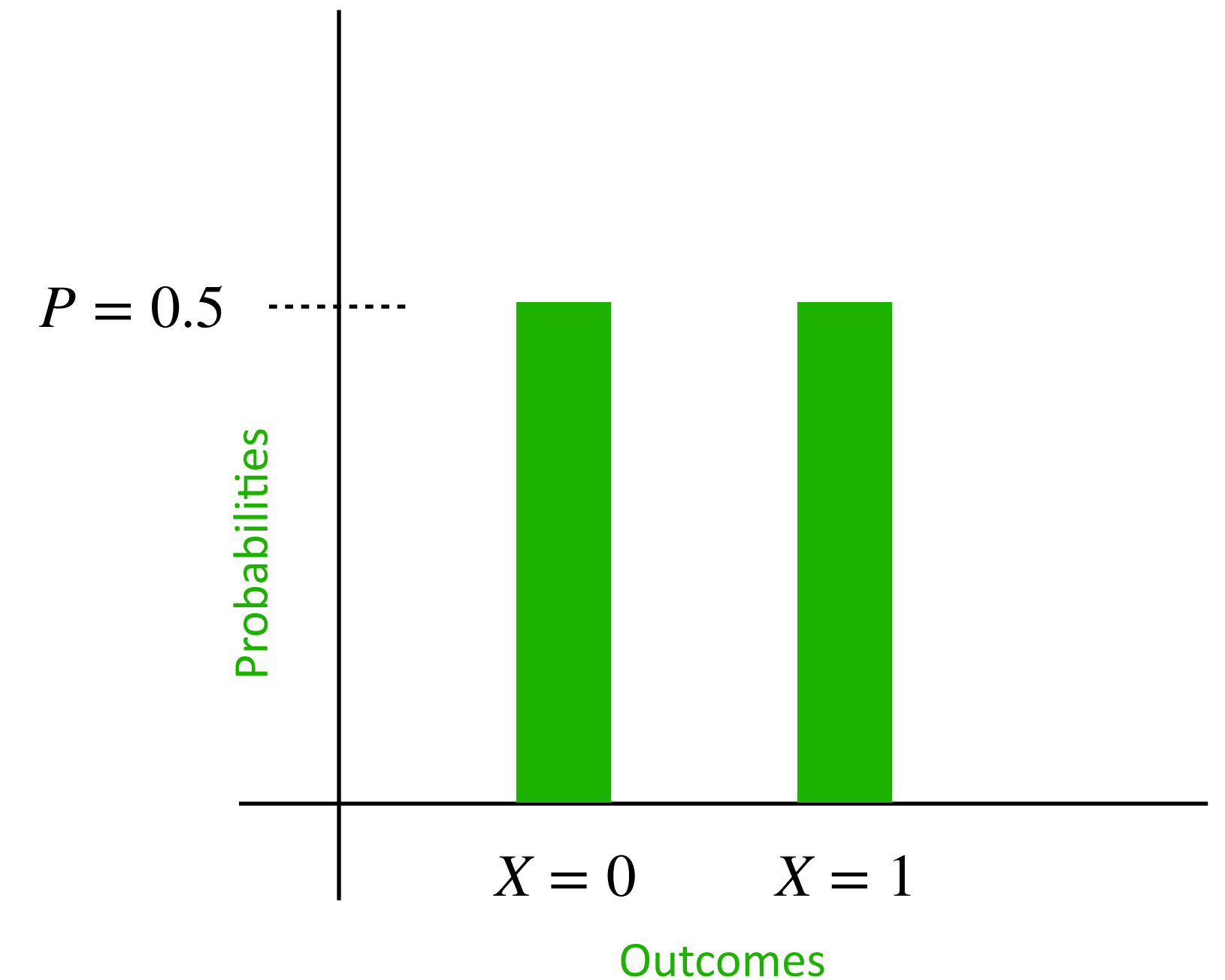
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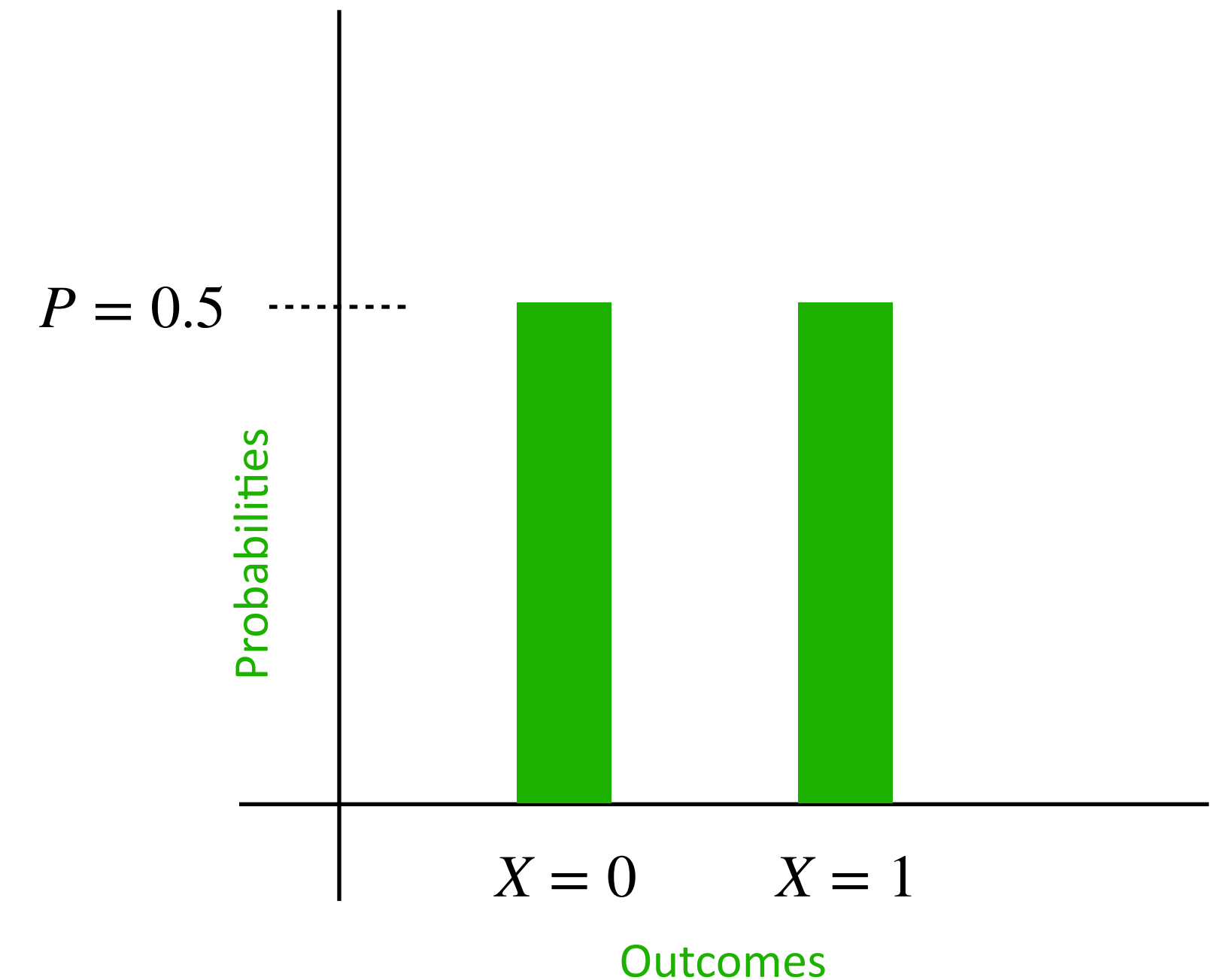
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This is the Probability Mass Function

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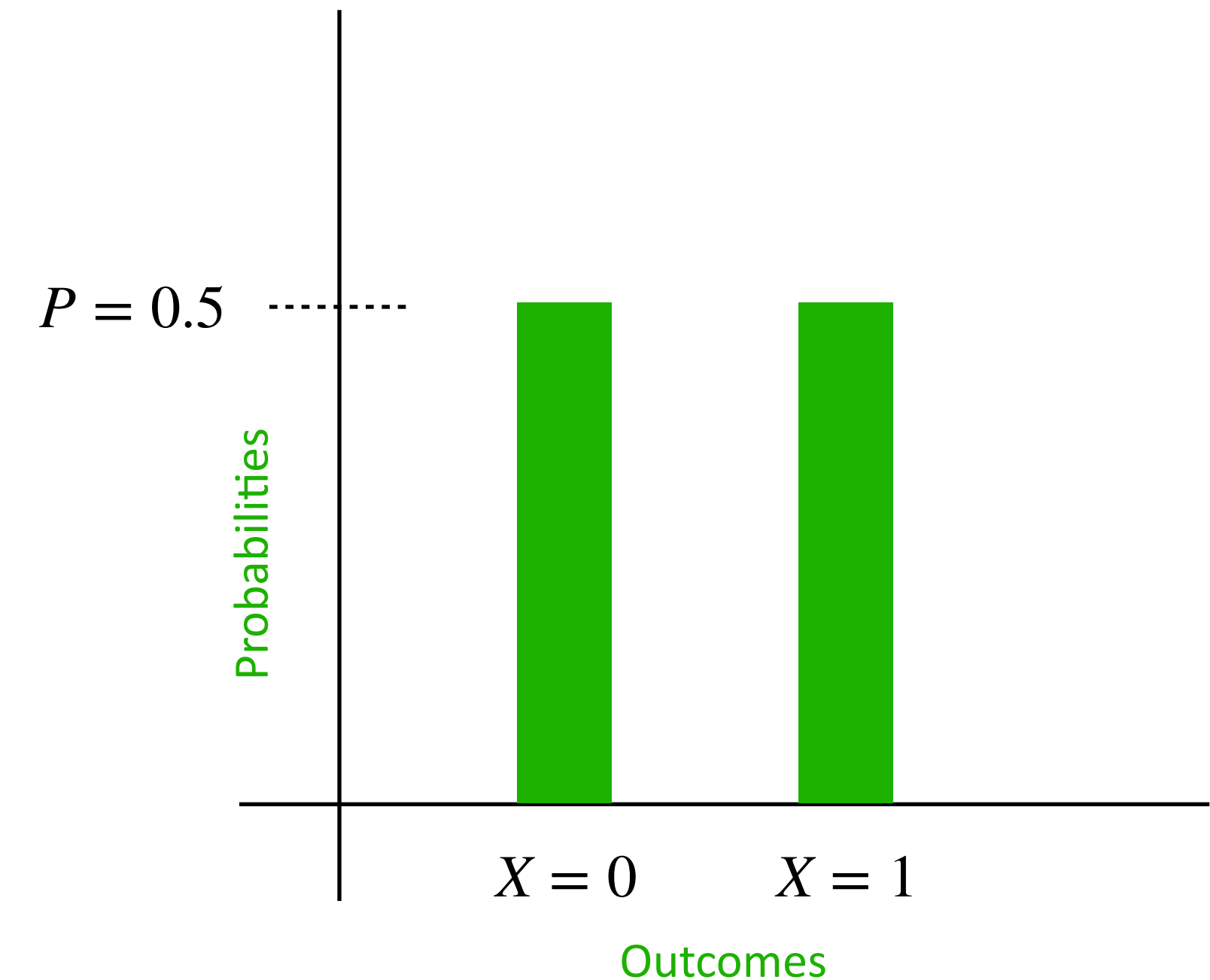
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The Probability Mass Function gives the probability that a discrete random variable is equal to some value

Probability Mass Function

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Mathematical Representation

Rolling a six sided fair die

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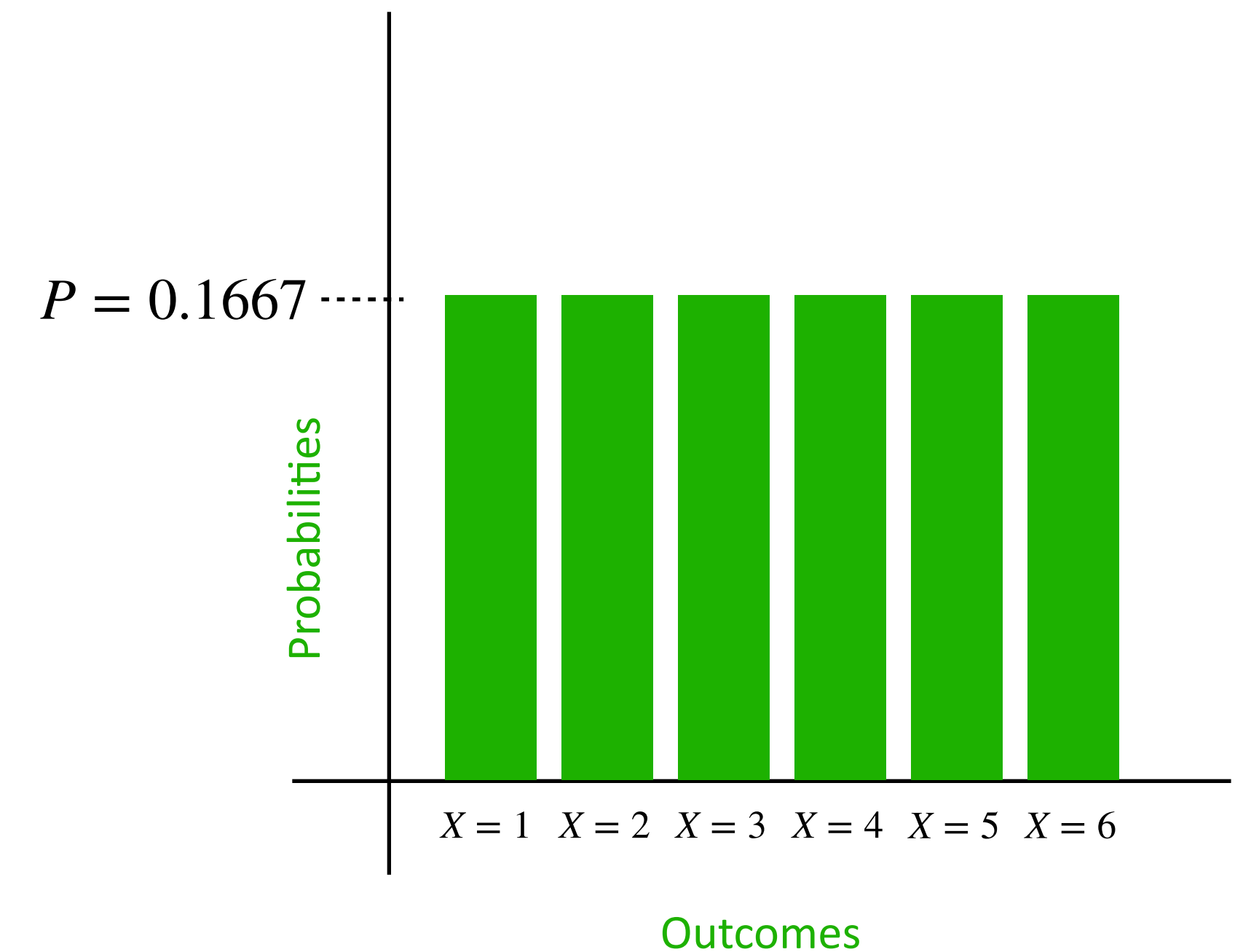
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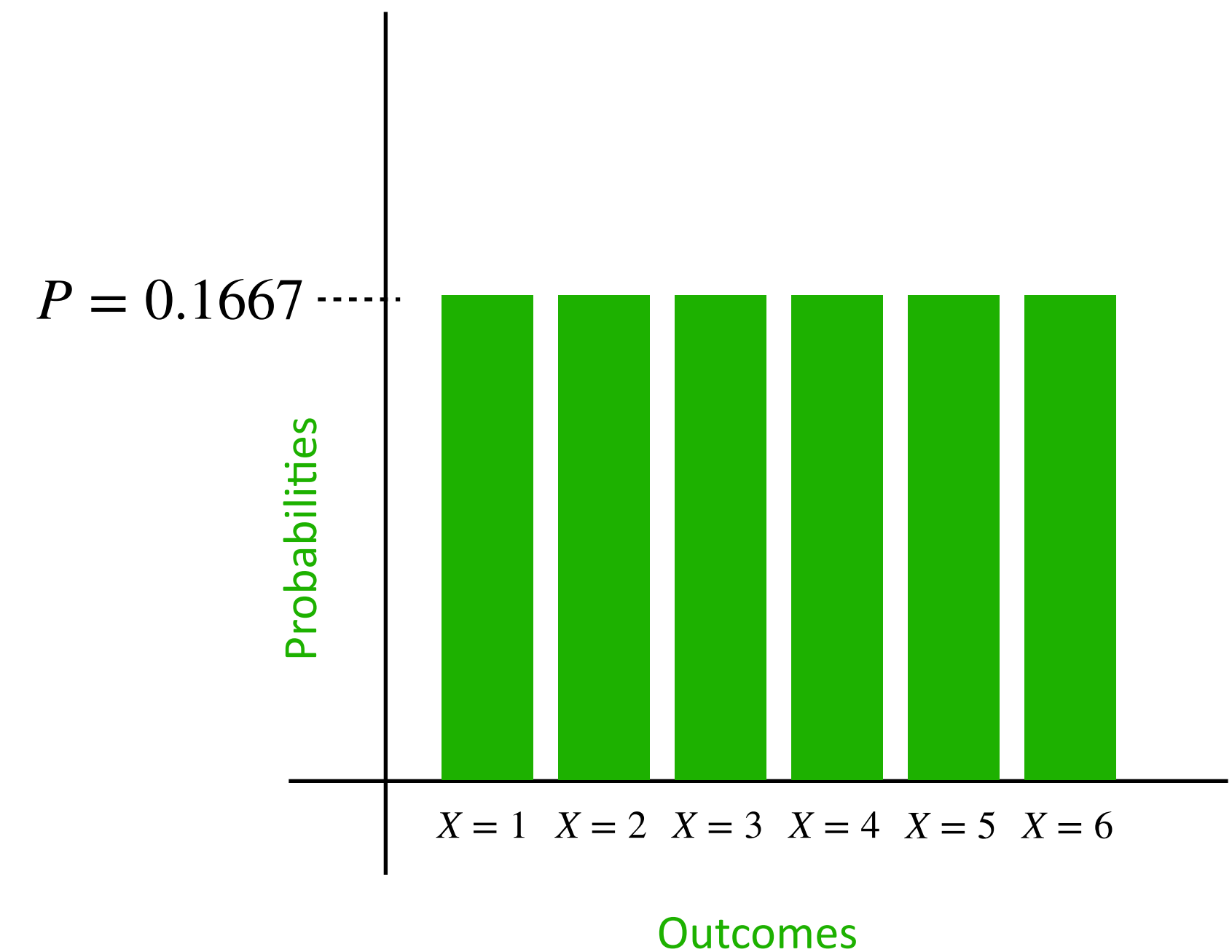
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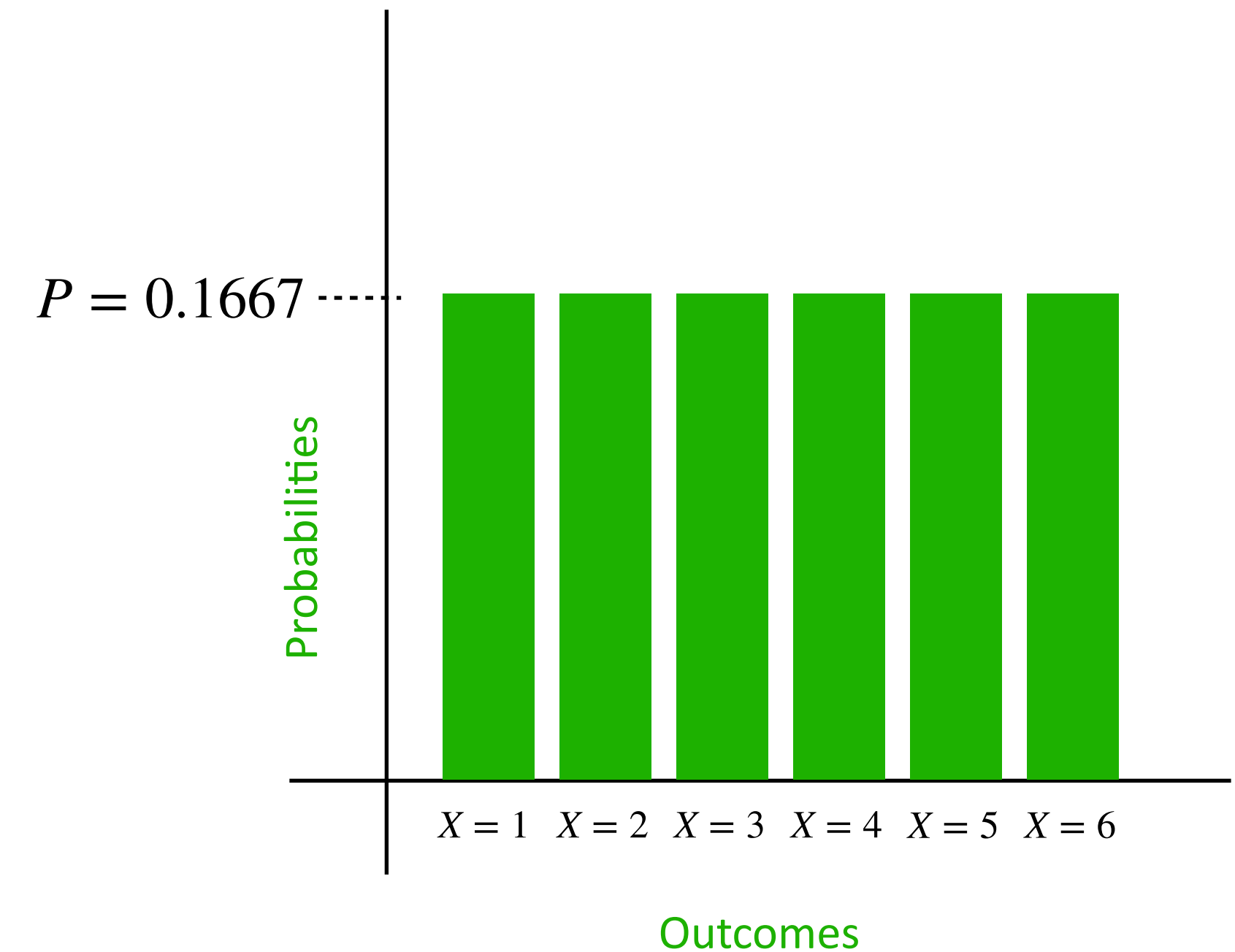
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Mathematical Representation

Probability Mass Function

Rolling a six

$$X(\alpha) = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{cases}$$

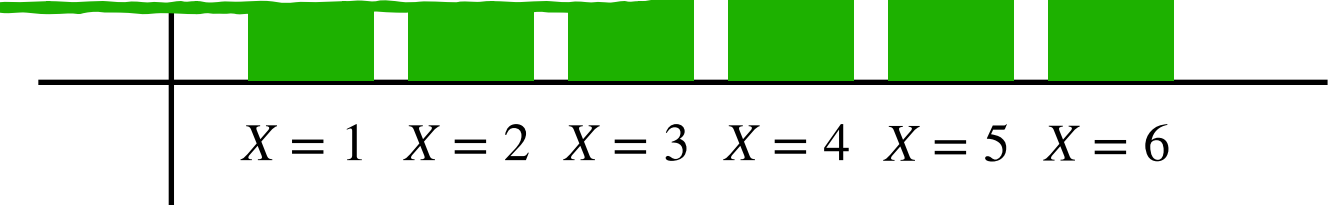
5, if $\alpha = \text{Five}$

6, if $\alpha = \text{Six}$

The Probabilities must
all add up to one

0.1667, if $x = 5$

0.1667, if $x = 6$



Outcomes

The Probability Mass Function gives the probability that a discrete random variable is equal to some value

Cumulative Distribution Function

Mathematical Representation

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Probability of rolling a 2 or less:

$$P(X = 1) + P(X = 2) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} = 0.3333$$

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Probability of rolling a 6 or less:

$$P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

Cumulative Distribution Function

Mathematical Representation

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Probability Mass Function

Rolling a six sided fair die

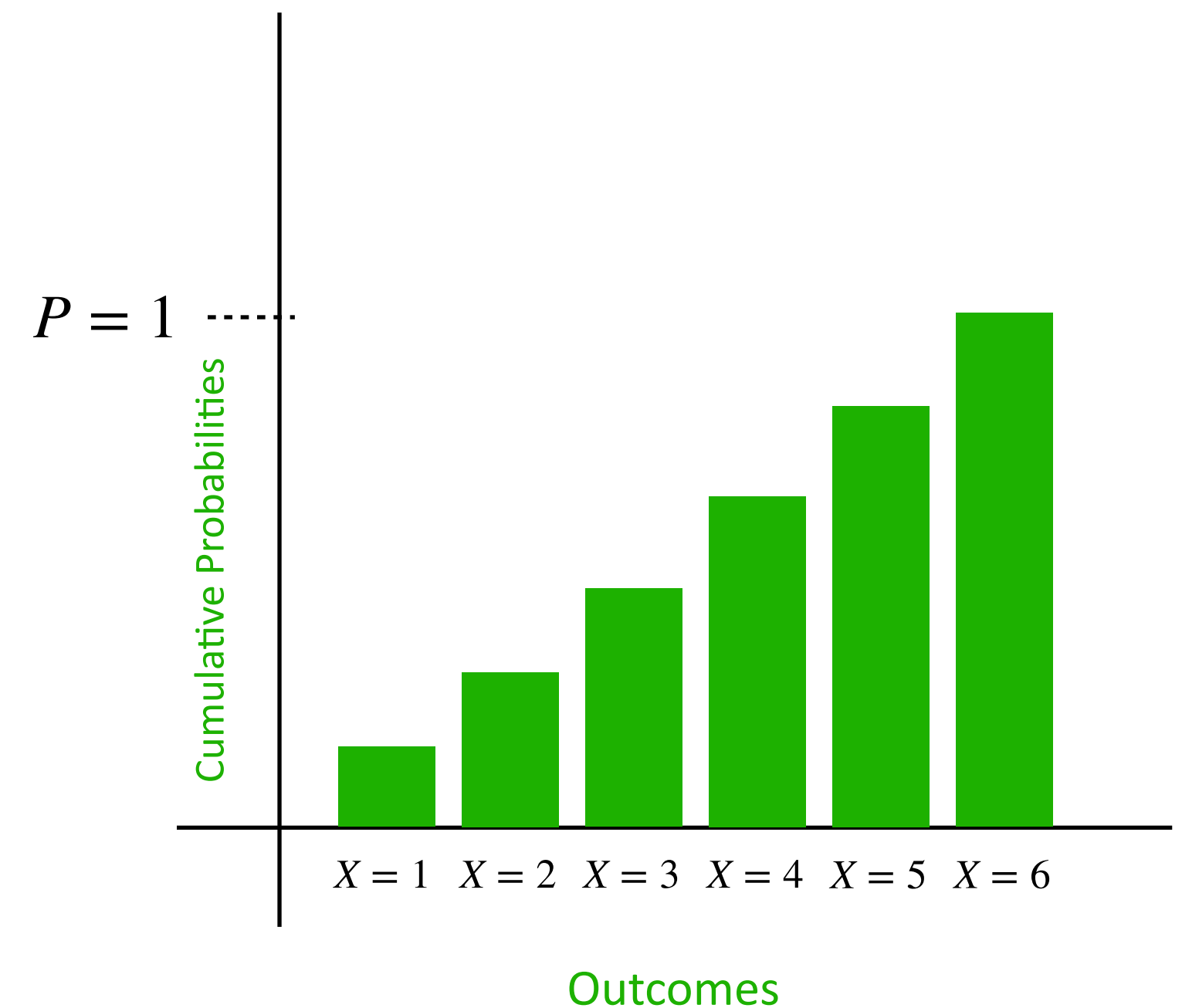
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Continuous Random Variable

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A random variable with an infinite, uncountable set of outcomes

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Example: X is the height of a citizen of Bulgaria

Infinite Outcomes: Height can be any value from 0 to infinity.

Continuous Random Variable

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Infinite Outcomes: Height can be any value from 0 to infinity.

A Continuous Random Variable doesn't have a Probability Mass Function because there are infinite outcomes.

Instead, a continuous random variable has a Probability Density Function (PDF)

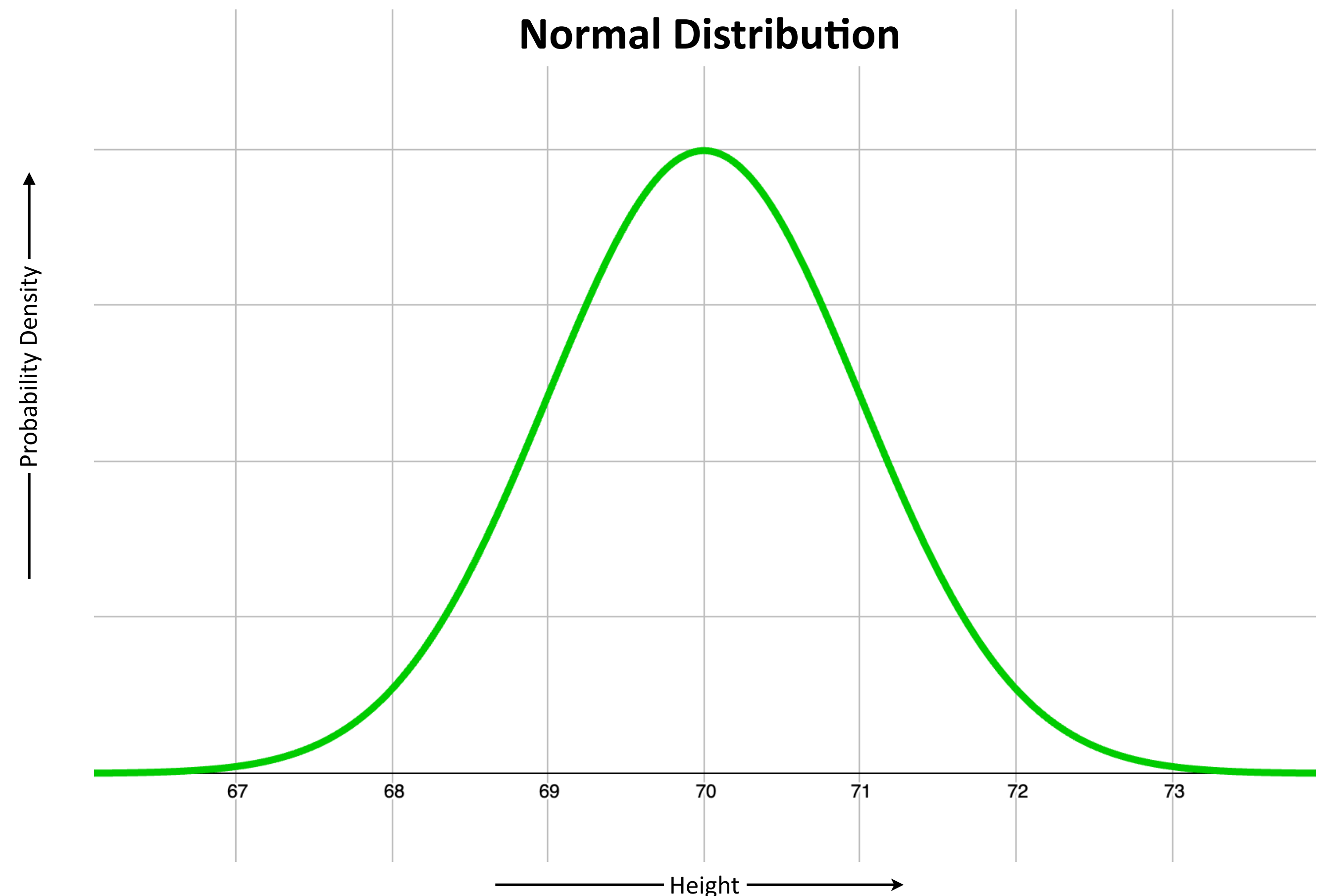
Probability Density Function

Continuous Random Variable

A random variable with an infinite, uncountable set of outcomes

Example: X is the height of a citizen of Bulgaria

The Probability Density Function returns the probability of an outcome within a certain range



Probability Density Function

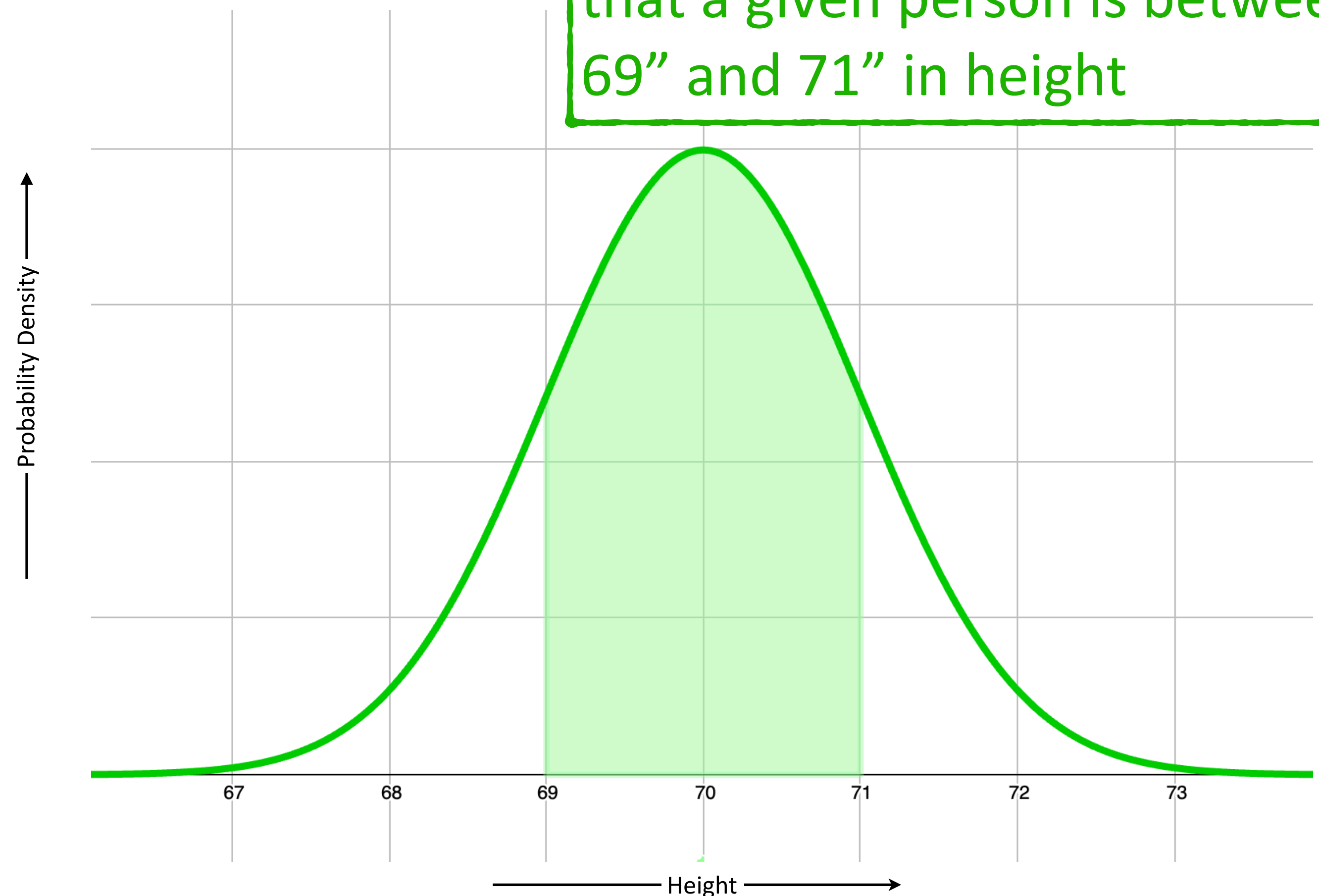
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The Probability Density Function returns the probability of an outcome within a certain range

Area under the curve: Probability that a given person is between 69" and 71" in height



Probability Density Function

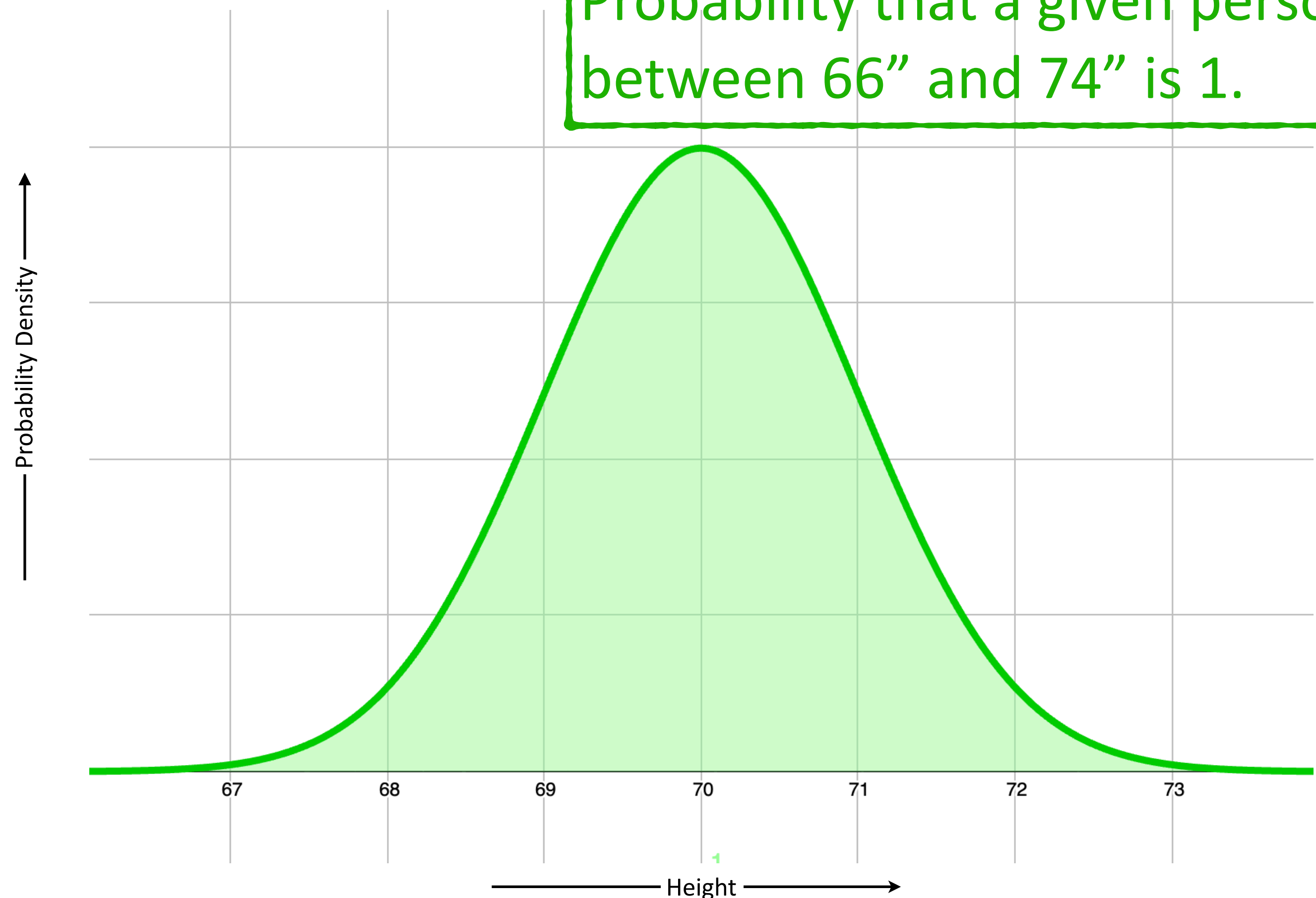
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The Probability Density Function returns the probability of an outcome within a certain range

Total Area under the curve:
Probability that a given person is between 66" and 74" is 1.



Probability Density Function

Continuous Random Variable

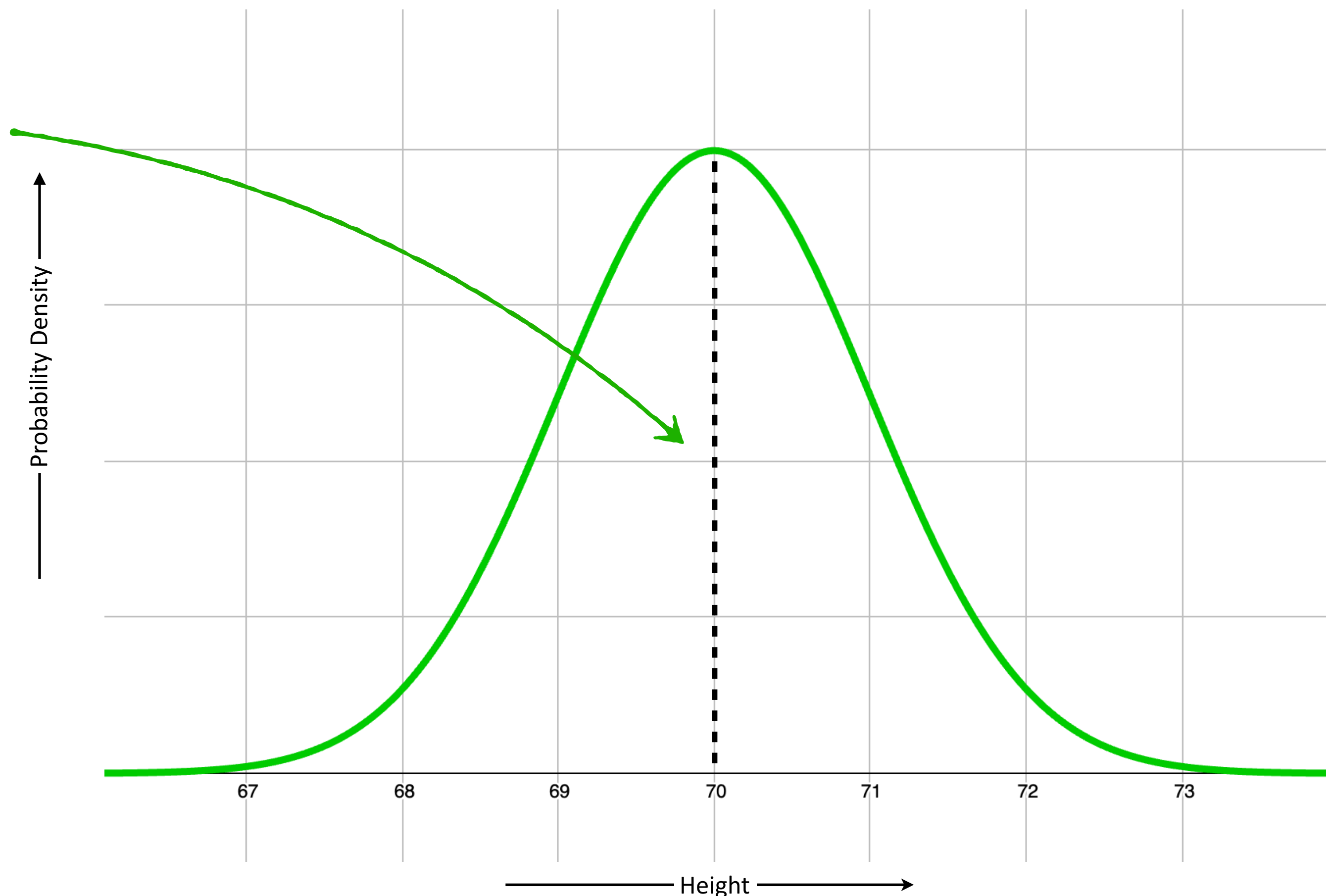
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Example: X is the height of a citizen of Bulgaria

Mean (μ) of the distribution

The Average Height of a citizen of Bulgaria is 70"

The Probability Density Function returns the probability of an outcome within a certain range



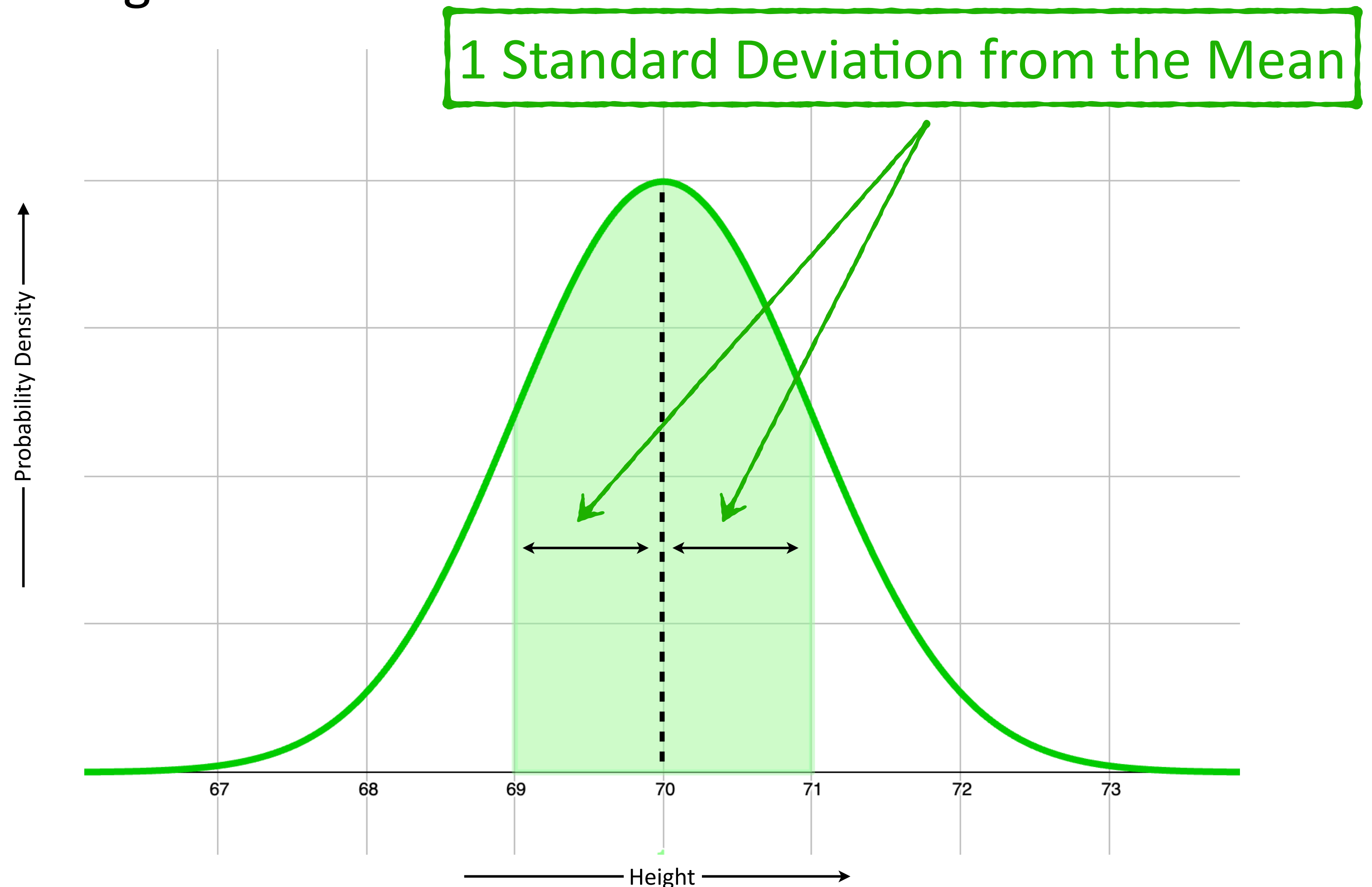
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Probability Density Function

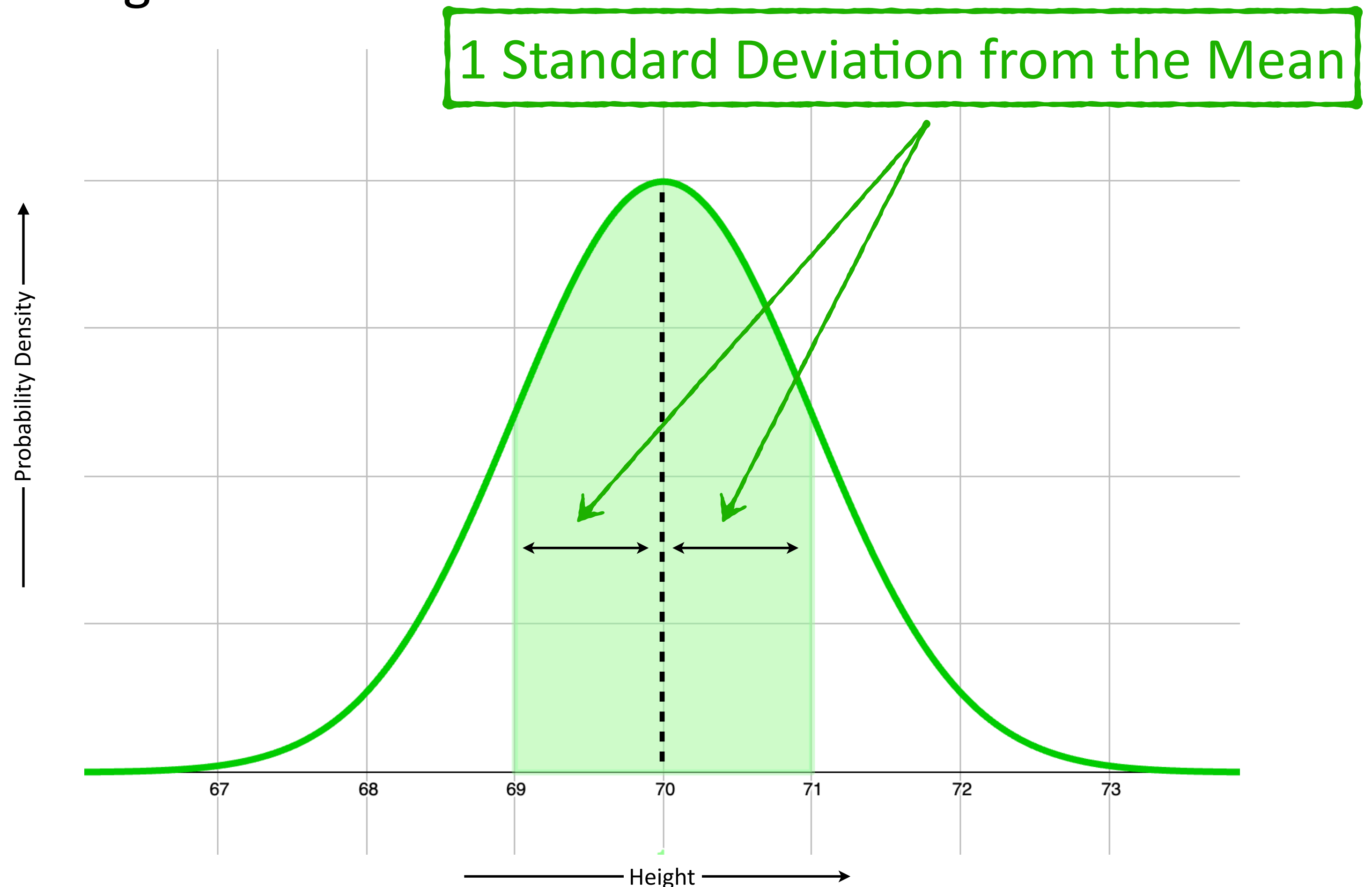
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Example: X is the height of a citizen of Bulgaria

Standard Deviation (σ): A measure of distance of the data from the mean.

The Probability Density Function returns the probability of an outcome within a certain range



Probability Density Function

Continuous Random Variable

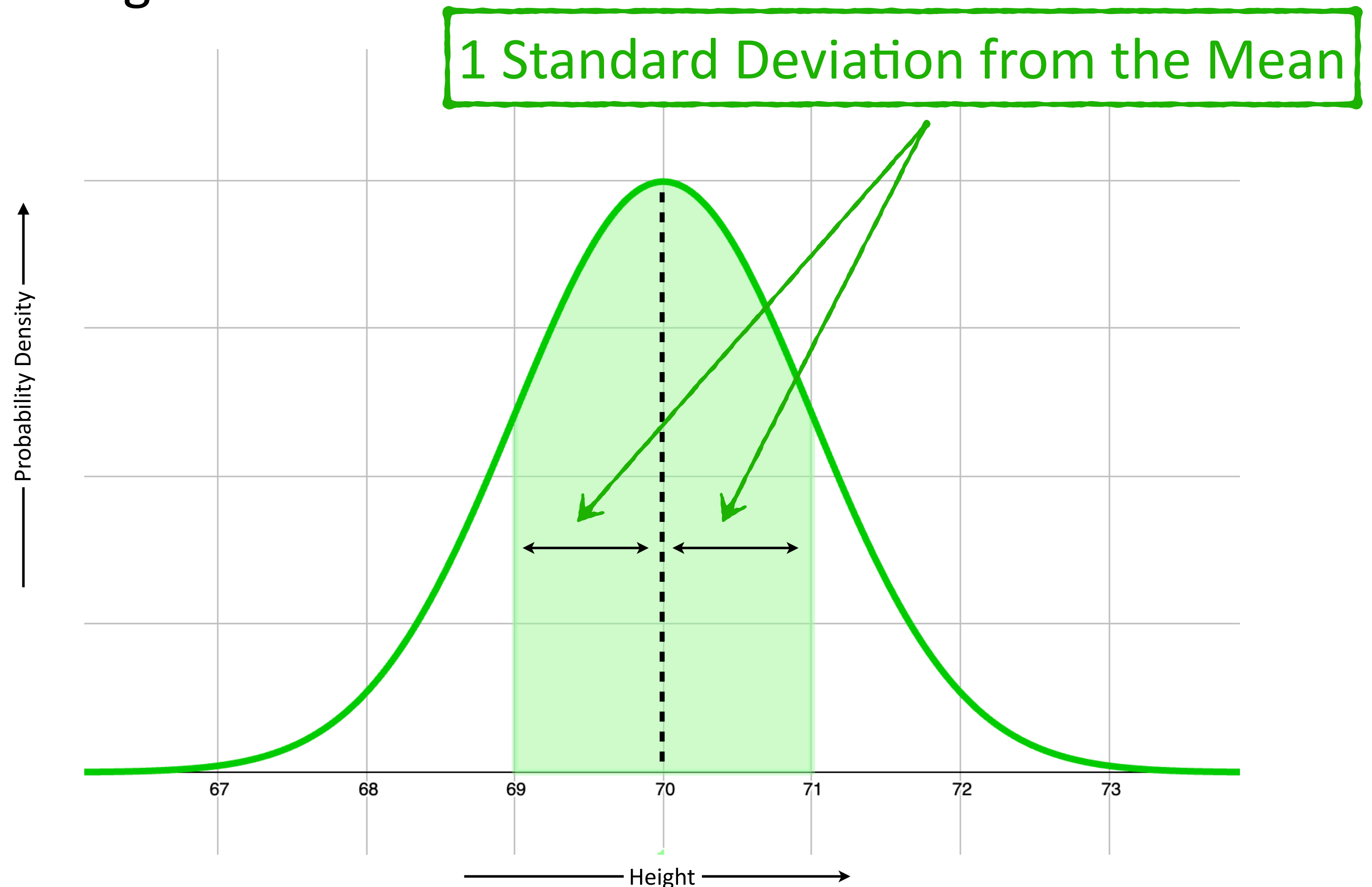
A random variable with an infinite, uncountable set of outcomes

Example: X is the height of a citizen of Bulgaria

Variance (σ^2) is the square of the Standard Deviation (σ)

Standard Deviation (σ): A measure of distance of the data from the mean.

The Probability Density Function returns the probability of an outcome within a certain range



Probability Density Function

Continuous Random Variable

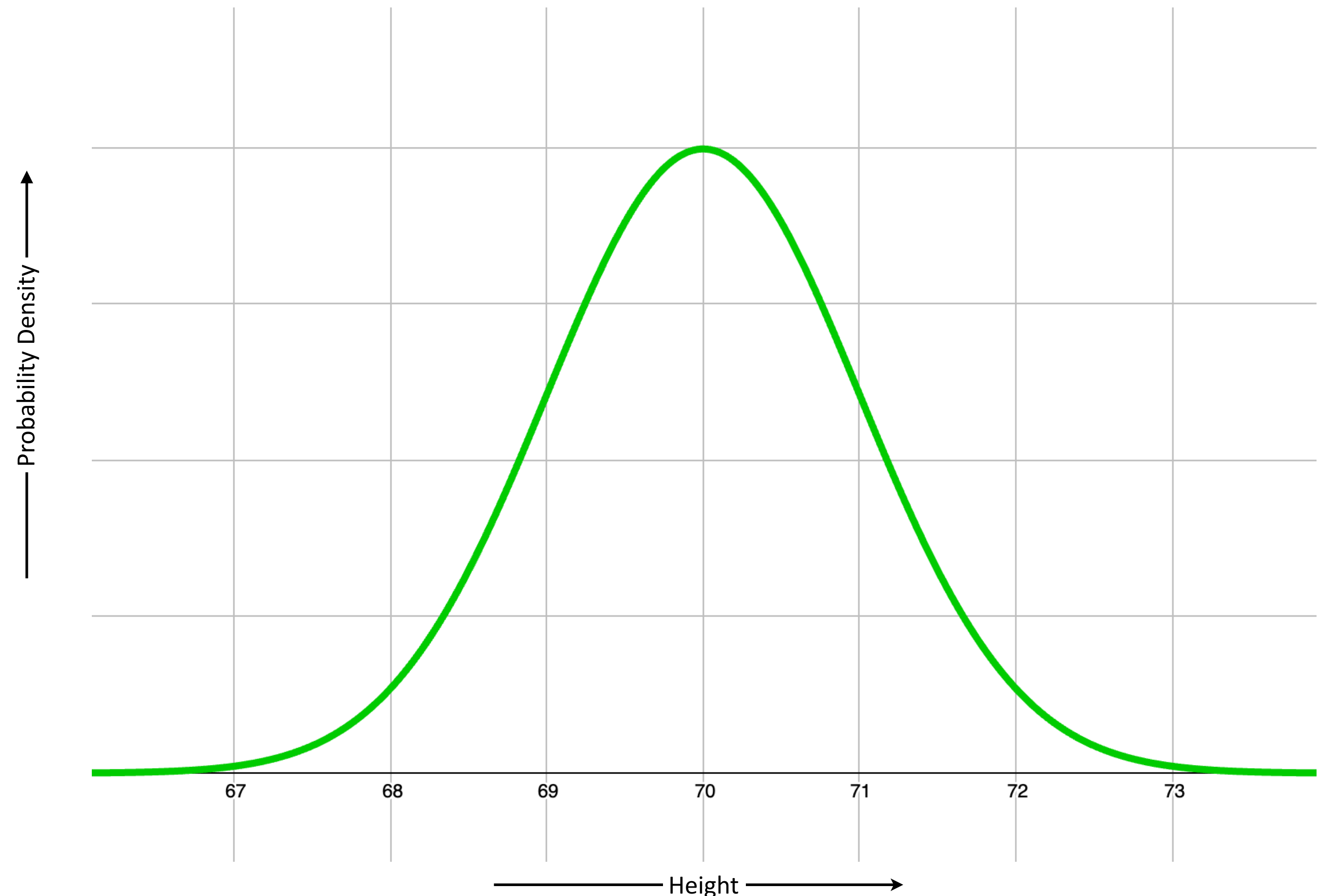
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Example: X is the height of a citizen of Bulgaria

Mean = μ

Variance = σ^2

Mean & Variance are the two parameters that determine the shape and position the normal distribution



Probability Density Function

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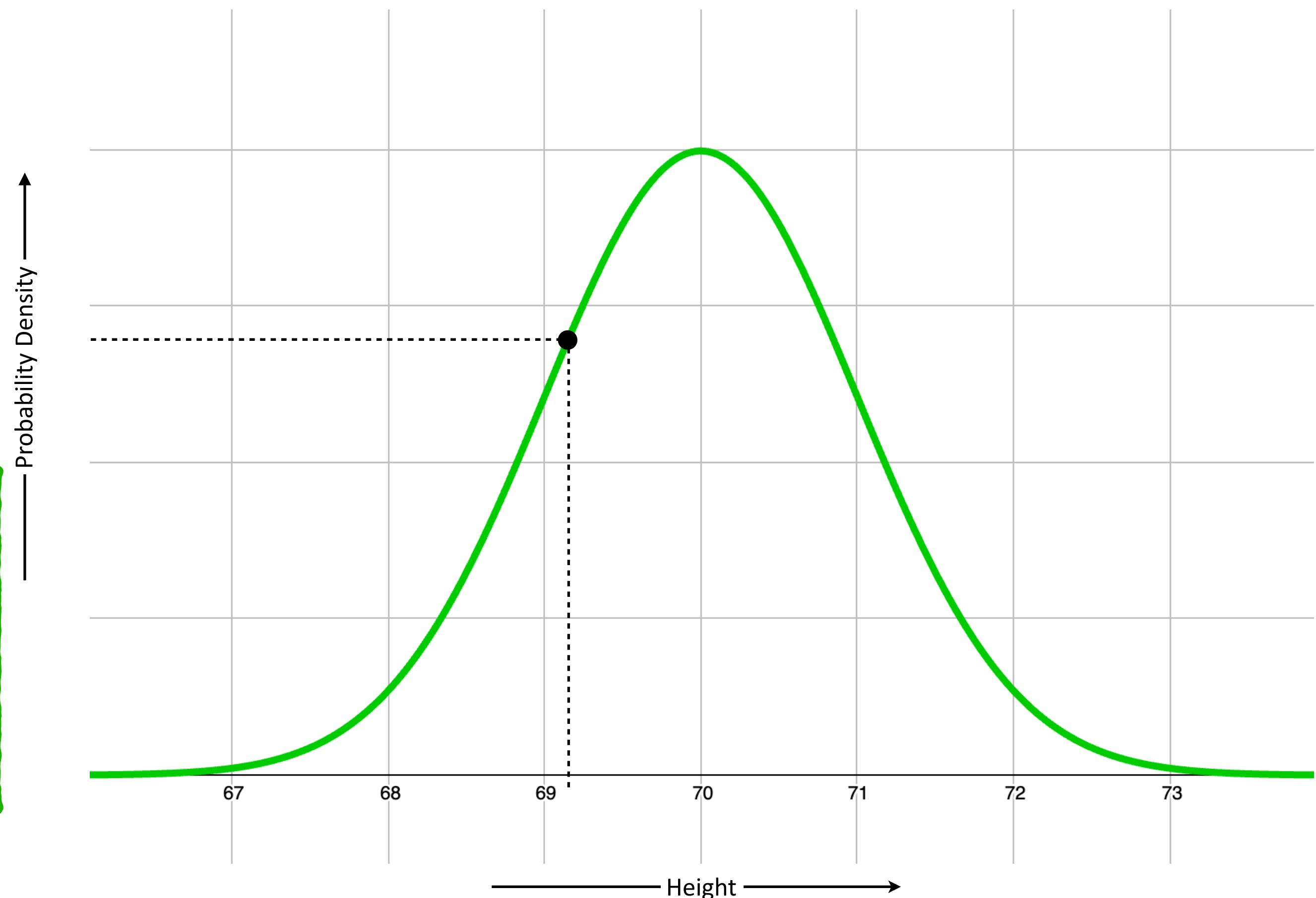
Example: X is the height of a citizen of Bulgaria

Mean = μ

Variance = σ^2

Likelihood is not Probability

A single point on the curve:
Likelihood of observing a person of that specific height for this specific distribution ($\mu = 70, \sigma^2 = 1$)



Probability Density Function

Continuous Random Variable

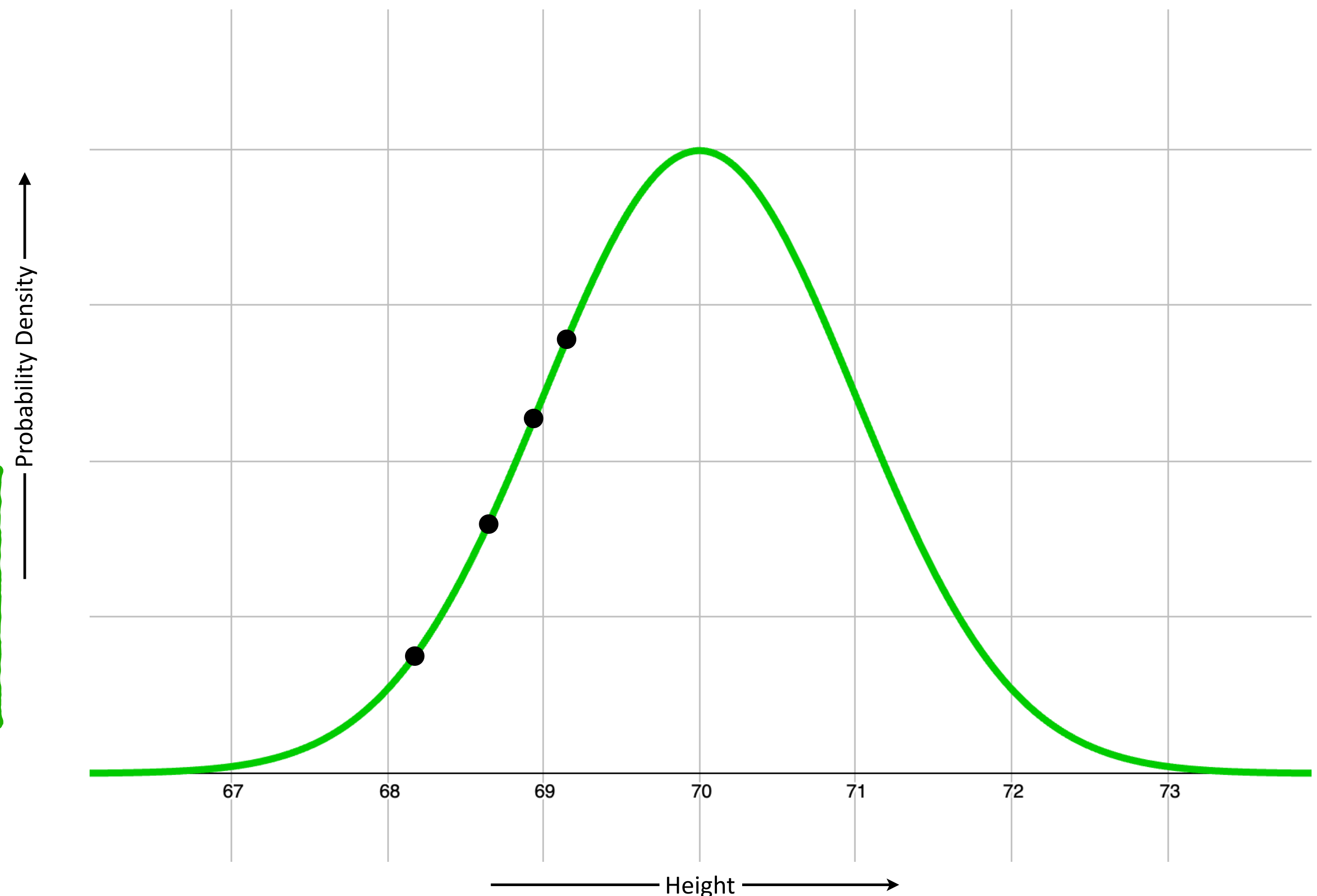
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Mean = μ

Variance = σ^2

Multiple points on the curve:
Multiply the likelihoods of every point



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Example: X is the height of a citizen of Bulgaria

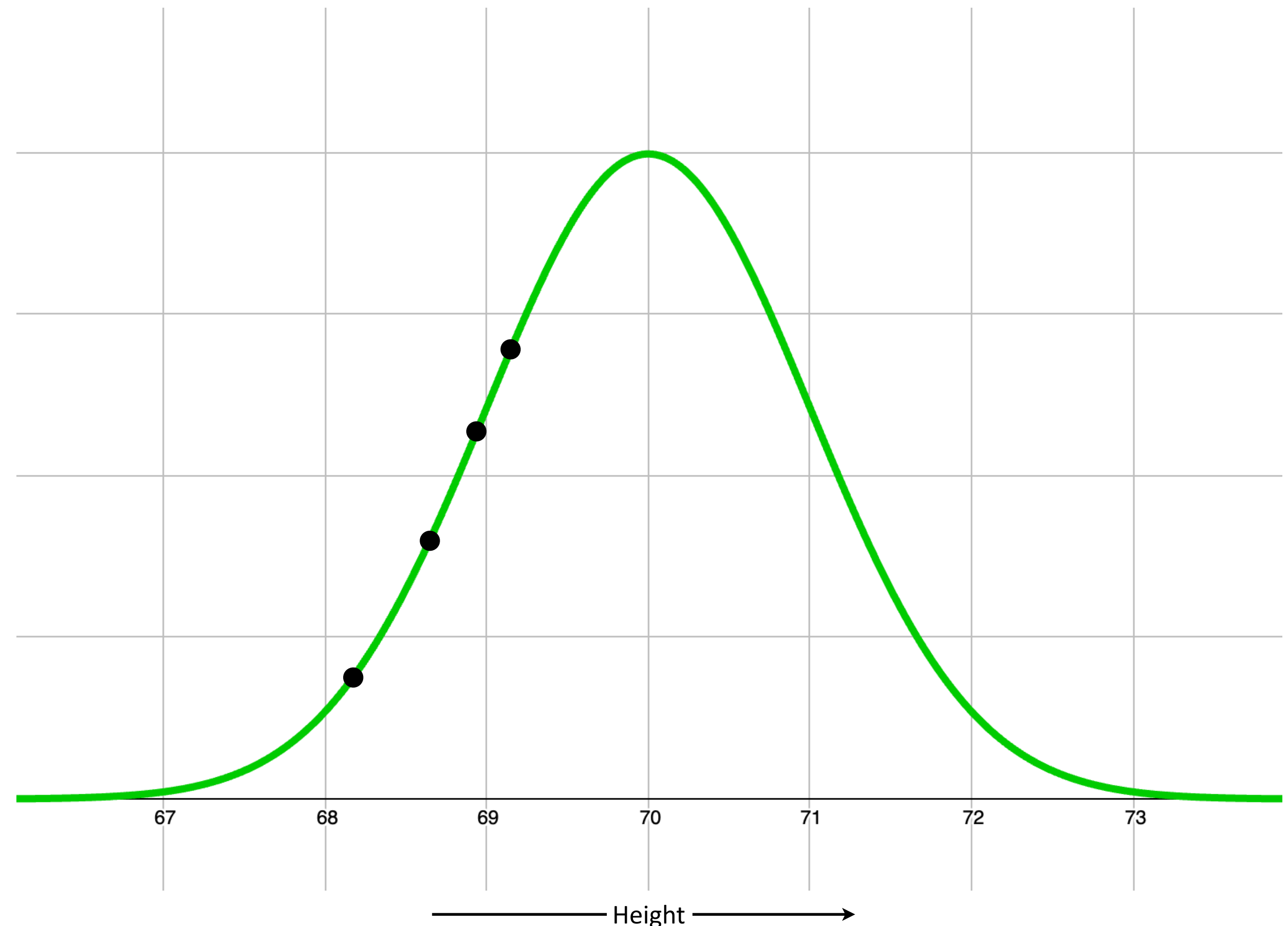
Mean = μ

Variance = σ^2

Log Likelihood

Multiple points on the curve:
Sum of the log of the likelihoods of every point

Probability Density



Probability Density Function

Continuous Random Variable

A random variable with an infinite, uncountable set of outcomes

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$f(x | \mu, \sigma^2)$

Probability Density function: Function that returns the probability of x within a range given fixed parameters μ and σ

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$$f(x | \mu, \sigma^2)$$

Likelihood Function

$$\mathcal{L}(\mu, \sigma^2 | x)$$

Probability Density function: Function that returns the probability of x within a range given fixed parameters μ and σ

Likelihood function: Function that calculates the plausibility of x taking a specific value for parameters μ and σ

Maximum Likelihood Estimation

Continuous Random Variable

A random variable with an infinite, uncountable set of outcomes

Example: X is the height of a citizen of Bulgaria

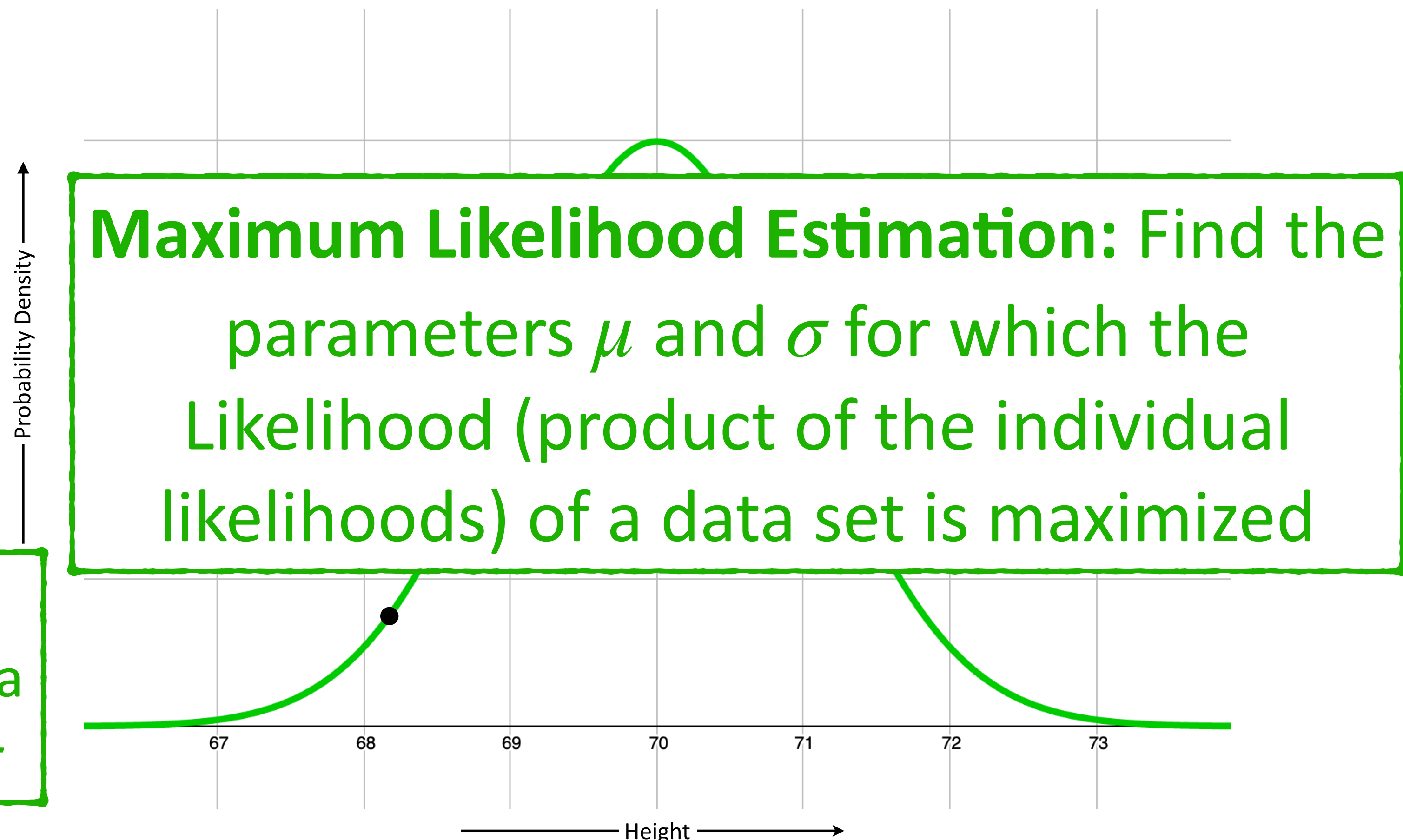
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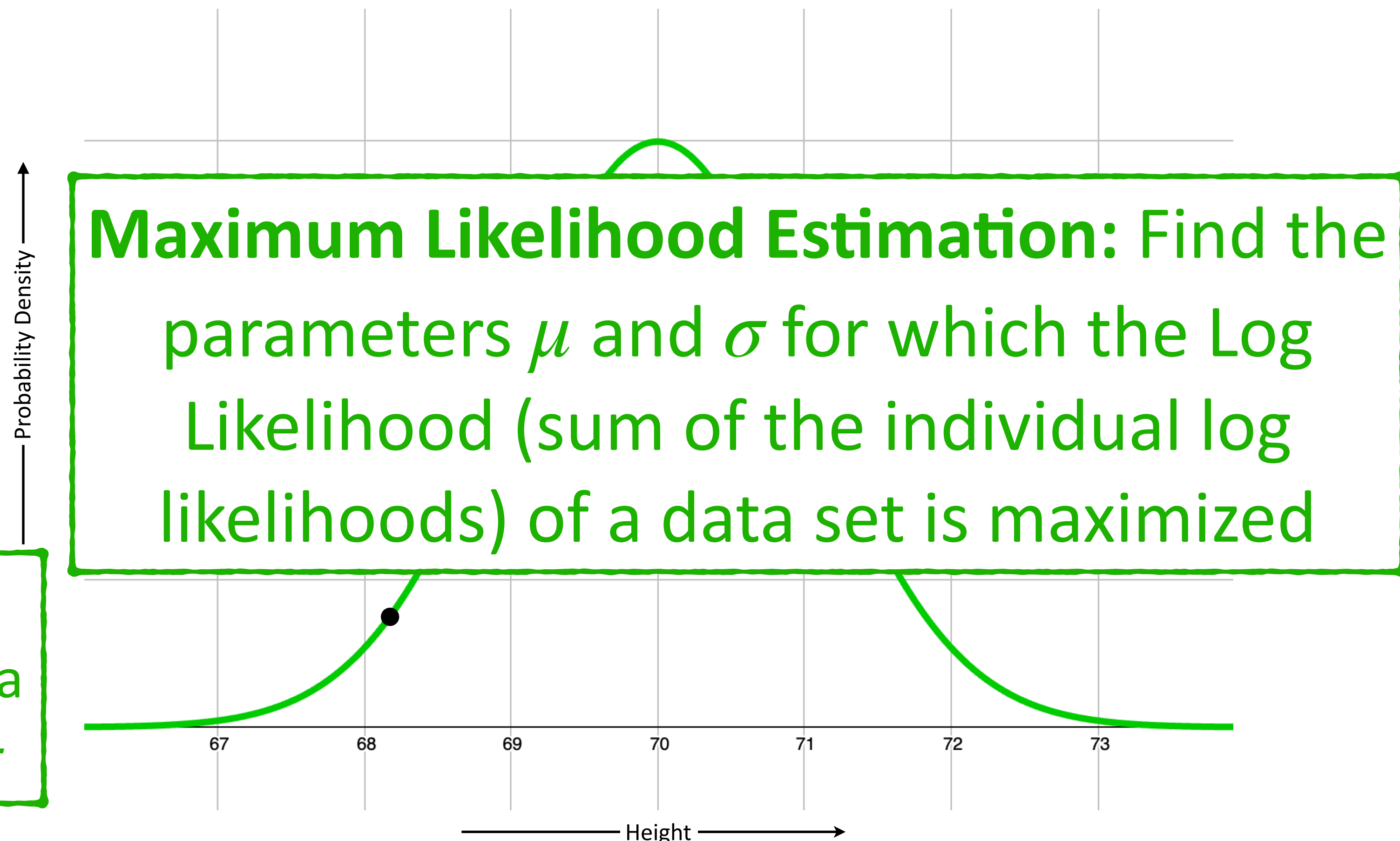
Mean = μ

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Likelihood Function

$$\mathcal{L}(\mu, \sigma^2 | x)$$

Likelihood function: Function that calculates the plausibility of x taking a specific value for parameters μ and σ



Lets review some well known probability distributions

Normal Distribution

Normal Distribution: The normal distribution is symmetric around the mean - outcomes near the mean occur more frequently than those further away from the mean. Also known as the Gaussian Distribution and characterized by the “Bell Curve”

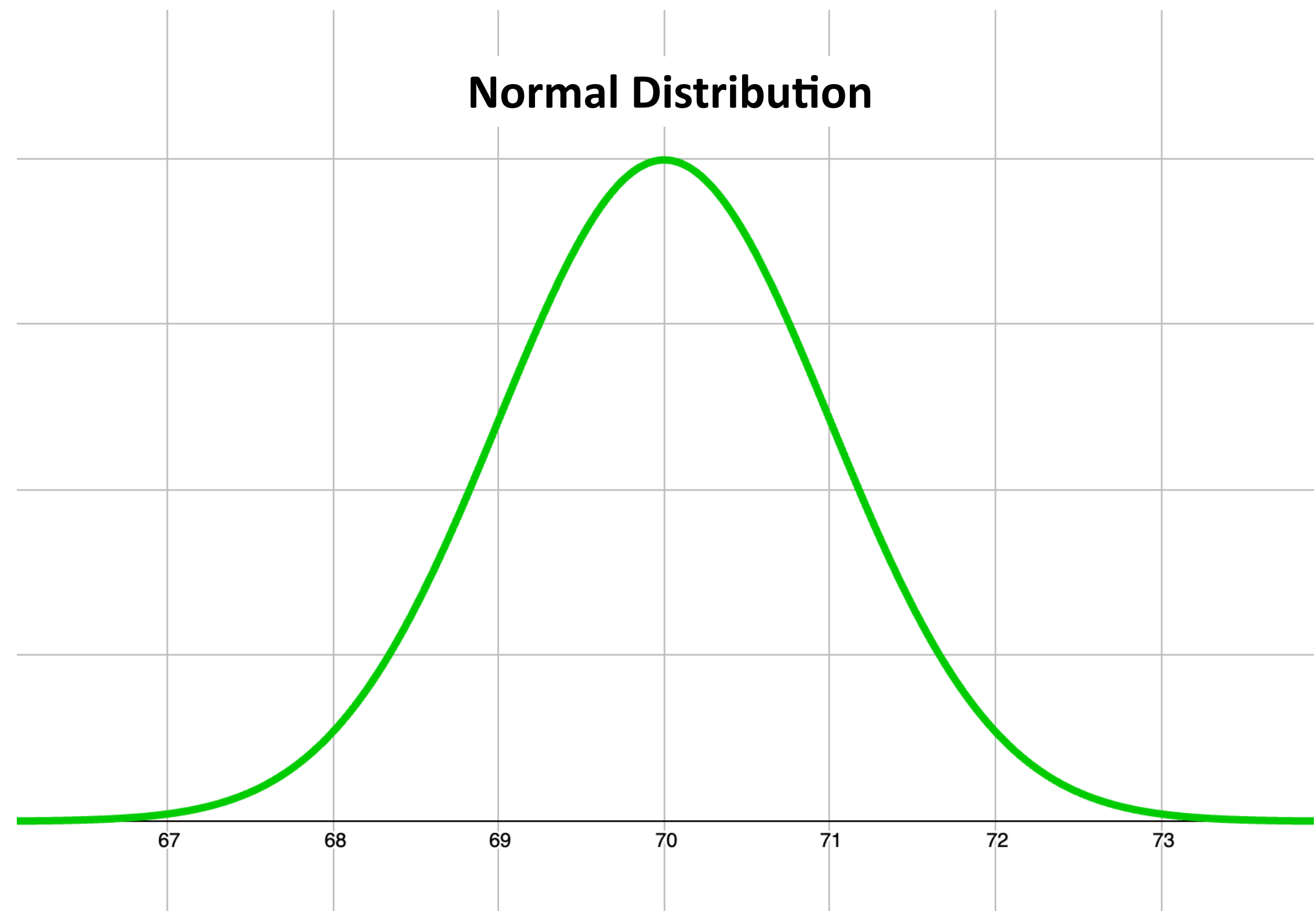
Parameters:

Mean = μ

Variance = σ^2

Probability Density Function (PDF)

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Binomial Distribution

Binomial Distribution: A Discrete Probability distribution of the number of successful outcomes in a sequence of n independent trials where each trial has binary outcome - true with probability p and false with a probability $1 - p$

Parameters:

Number of trials = n

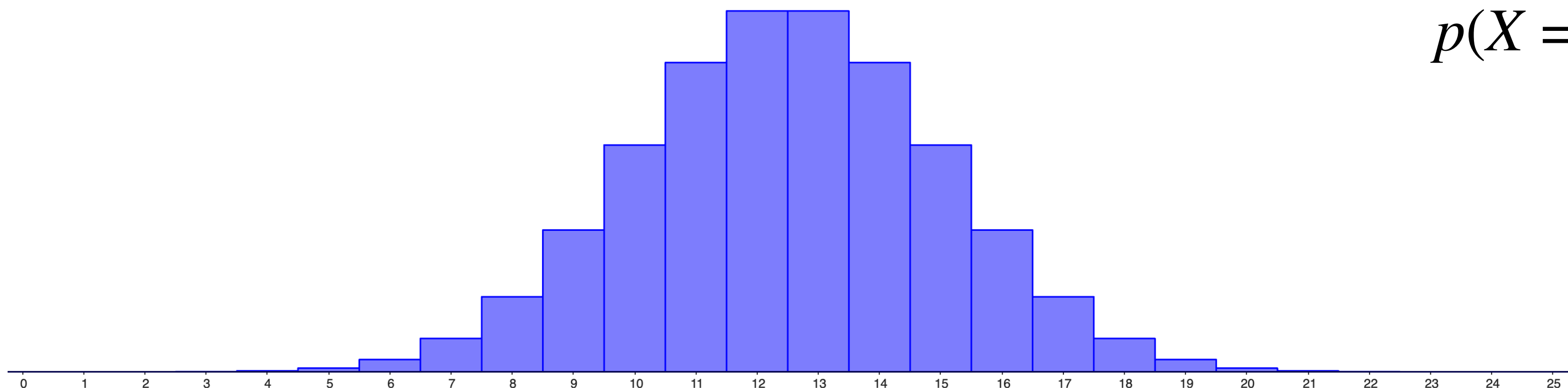
Probability of success of each trial = p

Probability Mass Function (PMF)

Probability of getting k successes in n independent Bernoulli trials

$$p(X = k) = \binom{n}{k} p^k (1 - p)^{(n-k)}$$

$$\binom{n}{k} = \frac{n!}{k!(n - k)!}$$



Binomial Distribution

$n = 25, p = 0.5$

Bernoulli Distribution

Bernoulli Distribution: A Discrete Probability distribution of a random variable that has an outcome of 1 with a probability p and an outcome of 0 with probability $1 - p$. Its a special case of the binomial distribution with $n = 1$ (a single trial)

Parameters:

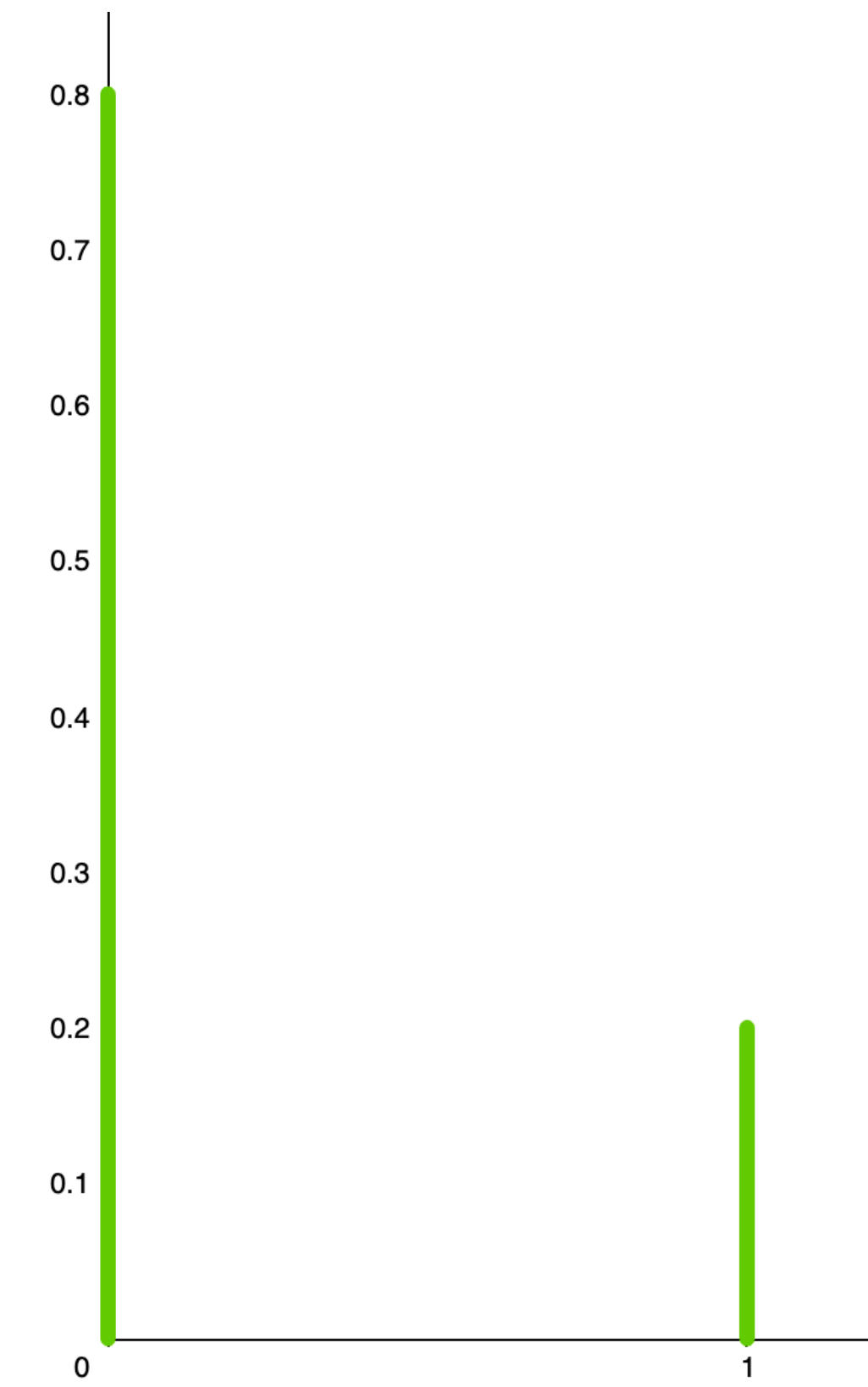
$$p(X = 1) = p$$

$$p(X = 0) = 1 - p$$

Probability Mass Function (PMF)

$$f(x) = \begin{cases} p, & \text{if } x = 1 \\ 1 - p, & \text{if } x = 0 \end{cases}$$

$$f(x) = p^k (1 - p)^{1-k}$$



Bernoulli Distribution

$n = 1, p = 0.2$

Probability vs Odds

Probability of an Event Occurring

The number of desired outcomes divided by the total number of outcomes

$$P(X) = \frac{\text{Number of Desired Outcomes}}{\text{Total Number of Outcomes}}$$

Odds of an Event Occurring

The Probability that the event will occur divided by the probability that the event will not occur

$$O(X) = \frac{p}{1 - p}$$

Related Tutorials & Textbooks

Logistic Regression ↗

An introduction to Logistic Regression. A Logistic Regression model use used to predict a binary value (the dependent variable) for one or more independent variables using a threshold to classify a probability.

Multiple Regression ↗

Multiple regression extends the two dimensional linear model introduced in Simple Linear Regression to $k + 1$ dimensions with one dependent variable, k independent variables and $k+1$ parameters.

Cost Function & Gradient Descent for Logistic Regression ↗

An introduction to the Cost function for Logistic Regression long with its partial derivative (the gradient vector). The model parameters (B & W) are then optimized using Maximum Likelihood Estimation and Gradient Descent.

For a complete list of tutorials see:

<https://arrsingh.com/ai-tutorials>